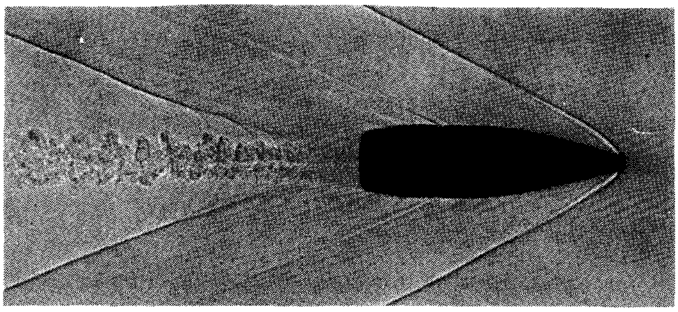
Battlefield Weapons Systems
& Technology, Volume X

MILITARY BALLISTICS



A Basic Manual

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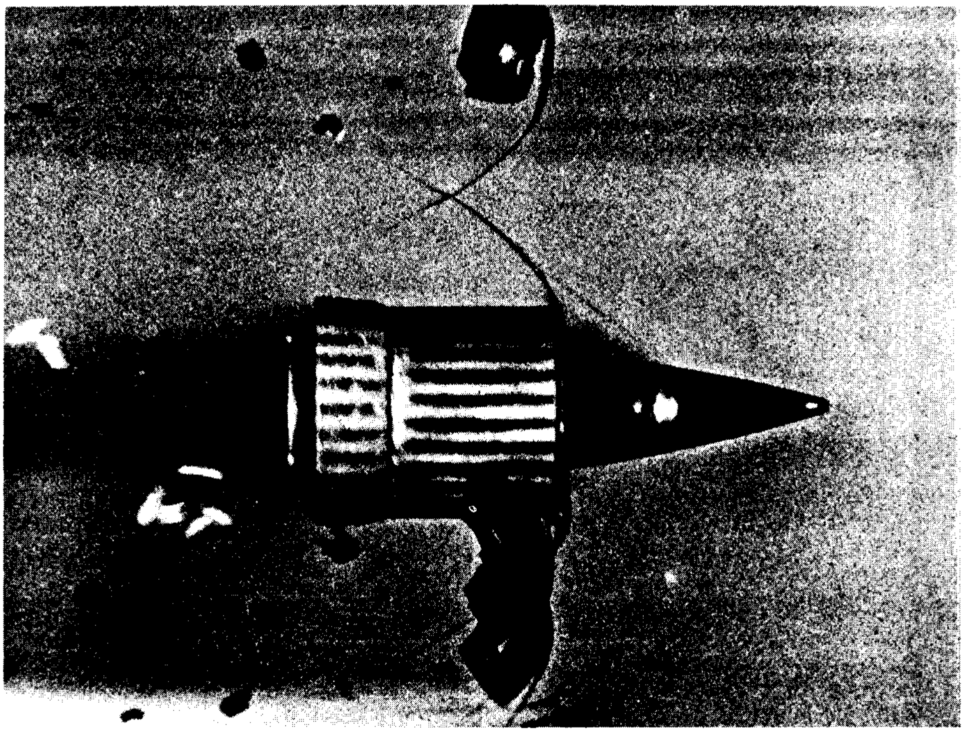
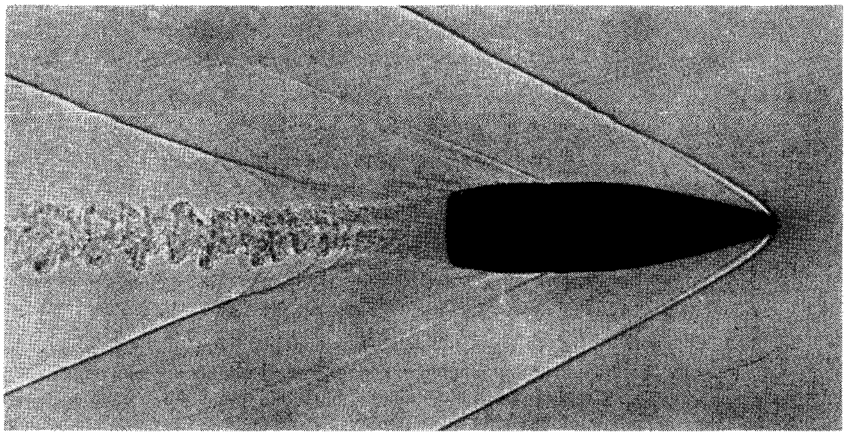


APDS showing discarded petals and separating from pot sabot

MILITARY BALLISTICS



A Basic Manual

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Preface

This series of books is written for those who wish to improve their knowledge of military weapons and equipment. It is equally relevant to professional soldiers, those involved in developing or producing military weapons or indeed anyone interested in the art of modern warfare.

All the texts are written in a way which assumes no mathematical knowledge and no more technical depth than would be gleaned from school days. It is intended that the books should be of particular interest to army officers who are studying for promotion examinations, furthering their knowledge at specialist arms schools or attending command and staff schools.

The authors of the books are all members of the staff of the Royal Military College of Science, Shrivenham, which is comprised of a unique blend of academic and military experts. They are not only leaders in the technology of their subjects, but are aware of what the military practitioner needs to know. It is difficult to imagine any group of persons more fitted to write about the application of technology to the battlefield.

This Volume

There are many textbooks on ballistics written primarily for the specialist. However, none of these offers a simple introduction to this complex subject. This volume concentrates on the principles of ballistics, illustrated by reference to military applications. The subject is broadly divided into its components of internal, intermediate, external and terminal ballistics. As the book is intended for use by both the army officer and scientist, some of the chapters are divided into two sections. The first part is largely qualitative while the second part provides a mathematical background for further study.

Shrivenham, November 1982 Geoffrey Lee

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Shrivenham
September 1982

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1.

History of Ballistics

EXTERNAL BALLISTICS

Pre-Newton Era

The first stone hurled by prehistoric man was probably the earliest example of external ballistics. The advantages of being able to throw farther and with more power led to devices such as slings and spears. Next came the bow, and an extension of it called the "ballista" from which ballistics derives its name. In turn, the word ballista owes its origin to a Greek word ballein, meaning "to throw". The ballista was a fairly complicated device used for propelling large arrows. (See Fig. 1.1).

It was the work of Leonardo da Vinci (1452-1519) which led to the early development of modern ordnance engineering. He designed many kinds of weapons, both offensive and defensive, ranging from cannon balls, mortars, rifled firearms, up to primitive versions of the tank and submarine. Da Vinci was also the first to provide a theoretical basis for the phenomena of aerodynamics; for example he conceived the idea of a centre of pressure whilst studying the flight of birds. External ballistics was founded as an exact science with the work of Galileo Galilei of Italy (1562-1642).

Galileo destroyed the Aristotelian theory of motion* and succeeded in laying the foundation for an accurate scientific study of motion. He deduced a parabolic path

*Aristotle reasoned that no inanimate body could move without a motive force; unless it belonged to four natural elements from which the Universe was made up; fire, air, water and earth. Air and fire could only move upwards, water and earth downwards, their natural homes. This could not explain the flight of an arrow, for example, after the motive force of man had been removed. To overcome this he introduced the medium in which motion takes place. This medium assisted all 'violent' motions.

for a projectile in vacuo. His pupil Evangelista Torrecelli formulated the equation for the range of a projectile, constructed the parabola and studied its various properties. The quadrant angles of elevation of cannon had been accurately measured from the time of Niccolo Tartaglia in 1537, but crude methods of measuring muzzle velocity were first found in the century following Galileo's death. It was then learned that the ranges actually attained by projectiles were very much shorter than those predicted by Galileo's parabolic trajectory. However, Galileo had known that the atmosphere resisted the motion of projectiles and thus he stated that his demonstrations were accurate "in the case of no resistance". Galileo argued that the retardation or acceleration due to the drag of the air on a moving body was a function "of weight, of velocity, and also of form". He stated that this resistance decreased with the projectile's density, increased with its speed and varied greatly with its shape. Galileo's work paved the way for Sir Isaac Newton (1642-1727) who was probably the greatest of the modern founders of ballistics. Newton's work on dynamics appeared in the "Philosophiae Naturalis Principia Mathematica".

The Principia contained two volumes: they were concerned with the motion of rigid bodies and the motion of fluids. Both subjects are of prime interest in modern ballistics. Newton began his argument on gravitation by considering the motion of a projectile fired horizontally from a mountain top in vacuo. He showed that, by continually increasing the initial speed of the projectile a speed would eventually be obtained which would result in the projectile moving completely around the earth and returning to pass through the firing position.

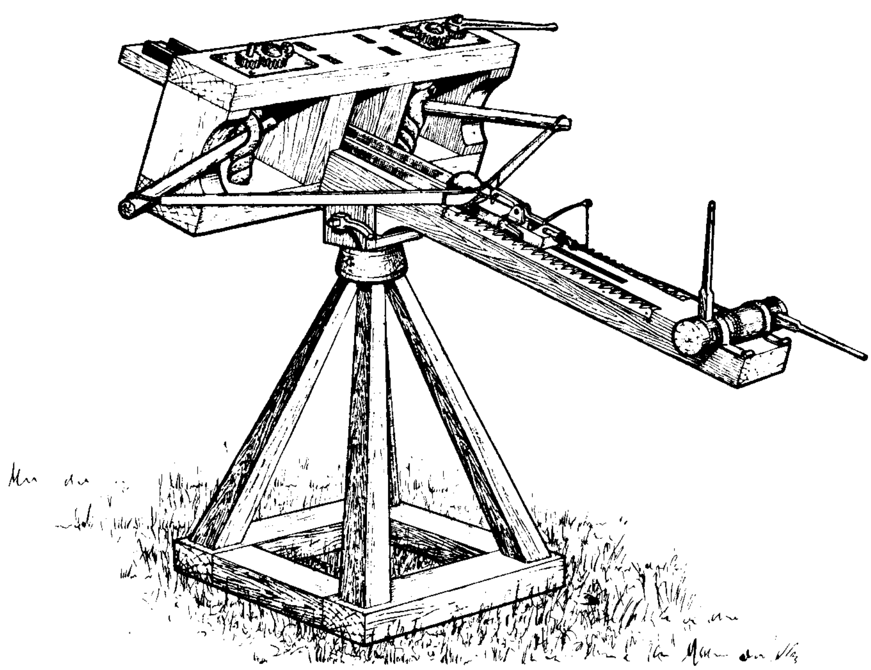

Fig. 1.1 Ballista

Post-Newton Era

The most important early successor of Newton in ballistics was Leonhard Euler of Switzerland (1707-1783). He analysed the results of experimental range firings in order to determine the drag on cannon balls. It is important to note that he was the first to work in analytical rather than geometrical terms.* In 1742 Benjamin Robins invented the ballistic pendulum and with it determined the muzzle velocity of musket balls. He was able to measure velocities up to 518 m/s at distances up to 76m from the gun. He investigated drag at low and high velocities, and found that Newton's velocity squared law for drag held quite well up to velocities of 244 m/s but that with velocities of 336 m/s or more the resistance was very much greater.+

The importance of accurate experimental methods for determining the drag of projectiles increased steadily during the 19th century because of developments in internal ballistics and ordnance which had increased both the magnitude and consistency of muzzle velocities. Progress in accurate timing of shell motion was made by Sir Charles Wheatstone (1802-1875) who used a screen through which the projectile passed, thus breaking an electrical circuit. This led to the work of Francis Bashforth; (1865-1870, 1878-1880) using an electrical chronograph, which he had devised for determining the drag of artillery projectiles.

Experimental work continued throughout Europe to provide a drag law, involving the velocity, for a standard shell. It was during the 19th century that a greater understanding of aerodynamics occured. Drag as a function of the properties of air was recognised and this gradually led to the shape of the projectile now in common use. The smoothbore guns of the 18th century produced low muzzle velocities and were inaccurate. This led to the appearance of the rifled gun in European warfare during the early part of the 19th century. Cannon used by European armies in the 18th century were effective only at short distances because of their low muzzle velocity and internal clearances. This led to the return of the artillery rocket.

Incendiary rockets had been extensively used in Italy and Germany during the 14th century, but they were gradually abandoned in European land warfare after 1450, largely because of their tendency to explode during manufacture or upon firing. Rockets continued to be used in the Middle East however. The Indian rockets were inaccurate and consisted basically of iron tubes weighing three to five kilograms. However, they were effective enough to alarm the British army campaigning in India, and they interested William Congreve of England (1772-1828) who developed incendiary rockets which could achieve ranges of about three kilometres. As a consequence, by the time of Napoleon every European nation had a Rocket Corps in its army.

*The letter e was first used to denote the base of the natural system of logarithms in one of Euler's papers on ballistics.

+We now recognise that this rise in resistance at approximately 340 m/s is due to the transition from subsonic to supersonic flight.

The spin-stabilised rocket was invented by the American, William Hale, in 1855. With the adoption of rifled artillery came the first need to treat the projectile as a body subject to aerodynamic forces other than gravity and drag. Ballisticians now had to explain other phenomena such as projectile drift. During the 20th century a thorough mathematical basis has been established to describe all the aerodynamic forces acting on a projectile in flight. In recent years the elaborate ballistic tables for calculating trajectories have been largely superseded by computers. Of course the drag law for individual projectiles is still required and may sometimes be deduced from theoretical considerations coupled with wind tunnel tests and live firings in a laboratory setting rather than the extensive range firings formerly undertaken.

INTERNAL BALLISTICS

The Rise of Gunpowder

The history of internal ballistics begins with the use of gunpowder and, although the actual date of its first use as a propellant has never been accurately determined, it dates back to the early 14th century and was certainly used in the battle between the English and French at Crecy in 1346. By the end of the 18th century the composition of gunpowder was fairly well standardised at 75 per cent saltpetre, 15 per cent charcoal and 10 per cent sulphur. The first recorded attempt to test powder was made by Bourne in 1578. He fired the powder in a small metal cylinder and the extent to which the lid rose gave an indication of the "strength" of the powder.

Early Measurements

The earliest attempts to measure the ballistics of the powder were made by Luys Collado in Italy and William Eldred and Nathanial Nye in England, during the 17th century. These attempts consisted of firing shots at a series of elevations and measuring the range. Benjamin Robins with his invention of the ballistic pendulum in 1742 measured the muzzle velocity of musket balls and deduced the pressure of the propellant gases. His book "New Principles of Gunnery" published in 1742 dealt with the fundamental problem of internal ballistics namely, achieving a given velocity within the pressure limits. The first attempt to measure the pressure of propellant gases directly was made by Count Rumford in America in 1792. His experiments led him to deduce a relation between pressure and density of the gases. At the end of the 18th century ballisticians were able to calculate the relation between pressure and shot travel, using Rumford's pressure density relation and assuming that the charge was completely burnt before the shot started to move. Integrating the pressure-shot travel curve enabled them to calculate the muzzle velocity and so to compare their results with experimental findings.

Piobert of France announced his laws of burning in 1839. Although these laws relate to black powder, one of them, namely, that the burning of each individual granule takes place in parallel layers, has been found to be applicable to modern propellants. Piobert also gave an approximate solution to the problem of the

motion of the gases in the bore, originally treated by Lagrange during the period of the French revolution. The approximate relation between the pressures on the breech and on the base of the shot was also worked out by Piobert. In 1857, General Rodman of America invented an "Indentation" pressure gauge for measuring the pressure of the propellant gases. The pressure was determined by the indentation made in a copper or lead plate by a wedged piston in contact with the gases. With this gauge he was able to measure the maximum pressure in guns and also deduced a pressure-density relation in a closed vessel. Rodman also showed how the shape of the granule affects the rate of burning.

The Basis of Modern Measuring Systems

A more accurate pressure measuring device called the "Crusher gauge" was invented by Andrew Noble in 1860. With this, he and Frederick Abel deduced the law relating to pressure and density at constant volume. The energy equation for burning propellants was given by Resal in 1864, thus laying the foundation of internal ballistics on thermodynamic principles.

By the end of World War II complex mathematical models existed. However these were generally inapplicable due to various simplifications. The advent of computers has enabled a more accurate determination of real gun systems and modern programs may have as many as fifty variables.

Modern Propellants

Modern propellants date from 1845 when a German chemist Christian Schonbein, discovered nitrocellulose, which burns completely leaving no solid residues. Gunpowder by comparison produces over half its weight as solid residue. A satisfactory propellant in the form of cakes was first produced in 1884 by a French physicist Paul Vieille by gelatinising nitrocellulose with an ether-alcohol mixture. This was used by the French army under the name of Poudre B. Alfred Nobel produced a similar propellant by using nitroglycerine instead of ether-alcohol. Abel in Britain gelatinised nitrocellulose by acetone using a mixture of nitroglycerine and vaseline. It was known as cordite due to its shape and was adopted by the British army in 1891 and is still used.

Since the firing of the first liquid propellant rocket by Robert Goddard in 1926, tremendous progress has been achieved in the development of liquid and hybrid propellants capable of reaching very high impulses. Prior to World War II, the use of solid propellant was generally restricted to armament rockets and boosters. Progress in this field has made its use in rockets and missiles widespread.

TERMINAL BALLISTICS

Armour Piercing Developments

The scientific study of the effects of a projectile striking a target is comparatively new. The early efforts to increase the efficiency of a weapon simply consisted of

making the shell larger. Introduction of armour and aircraft into warfare forced the development of armour-piercing devices. However, it is only the recent advances in metallurgy, strength of materials, and the development of sophisticated ballistic instruments capable of measuring very high pressure and phenomena taking place in the range of milliseconds that have enabled a proper study of terminal ballistics. Since the beginning of the Second World War considerable progress has been achieved in making high explosive shells and shots capable of causing heavy damage by blast, scabbing, fragmentation, and penetration. The study of the terminal effects of tactical nuclear weapons in recent years has extended the boundaries of terminal ballistics.

Forensic and Wound Ballistics

The use of handguns in civilian life has recently brought medicine and law into the realms of ballistics under the heading of Forensic ballistics. In parallel with it the damage caused by ammunition to the human body has given rise to a new specialisation called wound ballistics.

SELF TEST QUESTIONS

QUESTION 1 Who pioneered the early development of modern ordnance engineering?

Answer ....

QUESTION 2 Who succeeded in laying the foundation for an accurate scientific study of motion?

Answer ....

QUESTION 3 What two important subjects of prime interest to modern ballistics were contained in Newton's Principia?

Answer ....

....

QUESTION 4 What important invention was created in 1742 by Benjamin Robins to measure muzzle velocities of musket balls?

Answer ....

QUESTION 5 What important factor led to the shape of the projectile now in common use?

Answer ....

....

QUESTION 6 Why was rifled artillery adopted?

Answer ....

....

....

QUESTION 7 How did Bourne in 1578 attempt to test gunpowder?

Answer ....

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....

QUESTION 8 What important law did the Crusher gauge help to determine?

Answer ....

....

QUESTION 9 Until recently, what practical factors limited an extensive scientific study of the subject of terminal ballistics?

Answer ....

....

....

QUESTION 10 What are the two most recent applications of ballistics?

Answer ....

....

ANSWERS ON PAGE 201

2.

Internal Ballistics — Part I

DEFINITION OF THE GUN AND INTERNAL BALLISTICS

The gun can be viewed as a mechanical device in which heat, liberated by a burning propellant, is converted into the useful kinetic energy of a projectile, and its function is to propel projectiles toward specified targets. Internal ballistics is the scientific study of the operating processes within the gun from the moment that the burning of the propellant is initiated.

THE BALLISTIC COMPONENTS OF THE GUN

There is a wide variety of firearms to be found today; in practice the various types, from handguns through to heavy artillery, show distinct differences, but from a ballistic standpoint they are all similar. The majority of firearms work on the principle of the gun, and it is this majority which will be considered in this chapter. The few unconventional firearms, notably recoilless guns, will be just briefly described; they are in effect rocket powered and so do not conform to the traditional concept of the gun.

Firearms are devices for propelling projectiles toward specified targets: their common component is a tube in which both motion and direction are imparted to the projectiles fired. To avoid confusion, the gun will be defined here as having one closed end, whereas the recoilless gun is effectively open at both ends. The closed end of a gun forms a chamber which, when loaded, is filled with propellant and has a projectile positioned ahead of it. When the propellant is ignited it rapidly produces gases which in turn push the projectile along the barrel that forms the remaining part of the tube.

To fire a gun the propellant must be ignited, so an ignition device is included in the propellant chamber. Once ignited the propellant is said to burn, however it is unlike more familiar fuels as its combustion does not require oxygen from the air: only the chemicals within the propellant react to produce the resultant hot gases.

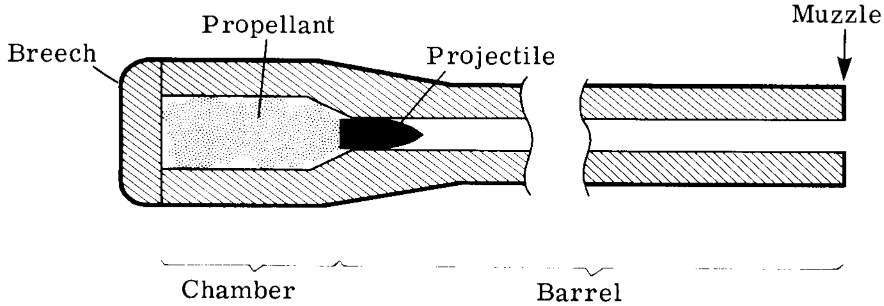


Fig. 2.1 The main ballistic components of the loaded gun

Many guns fire projectiles which need to be spun to maintain a stable flight. In such a case a gun has a rifled bore to its barrel, which causes the projectiles to rotate as they pass along the barrel.

The internal diameter of the barrel is called the calibre; if the barrel is rifled, the calibre is the diameter of the bore prior to the cutting of rifling grooves in the bore surface.

For ease of use the propellant and ignition device are often assembled as a unit in a combustible bag, or within a cartridge. Once inserted, a cartridge case can be considered as part of the chamber wall.

The components that are renewed for each firing of the gun are the projectile, the propellant charge, the primer and, where applicable, the cartridge case: they are collectively known as the ammunition. The dimensions and characteristics of these components together determine the loading conditions of the gun.

THE PRACTICAL GUN

The fundamental differences between the internal designs of guns are usually only differences in the size and the shape of the ballistic components. However, the definition so far is only sufficient to describe a simple muzzle-loaded cannon, so additional parts must be incorporated in the design to make the gun a practical and effective weapon. Four mechanisms must be included to enable mounting, loading, aiming and firing of the gun.

The simplest mounting is used for handguns and most rifles: they are simply hand held. Heavier guns of course need mechanical support and this is usually provided by a pivoting block at the chamber end which allows the gun to be turned for aiming.

Normally the back of the chamber can be opened so that a new projectile, propellant charge and primer can be loaded into the breech: this is achieved either manually or by an automatic mechanism. The designs of breech mechanisms vary significantly between the different types of guns, though their function is the same: it is to permit reloading of the gun for the next firing sequence.

Accurate aiming is achieved either visually by the use of sights or by calculated alignment of the gun according to data on the relative position of the target. An aiming device is of little concern in the study of internal ballistics, though the accuracy of both the aiming device and the internal ballistics are together vital to the gun's ability to deliver projectiles to the point of aim.

The final mechanism necessary to the operation of the gun is an ignition device. Usually this consists of a primer, which is a small but powerful propellant charge which on firing discharges hot gases and particles into the main propellant charge. The firing is initiated by electrical heating or mechanical crushing of the primer.

The internal ballistics of the gun that will be described in this chapter is only an idealised behaviour of the gun. It roughly describes the components, firing sequence and problems encountered in most guns. To model accurately the internal ballistics and response of specific guns, the effect of their added practical components must also be considered. The structure of a gun and its condition on firing have a most significant effect on the recoil of the gun and the flexure of its individual components. Internal ballistics alone cannot fully describe such effects, but it can provide explanations and predictions of the dominating forces and effects observed in most guns.

THE PROJECTILE

The projectile component of ammunition can also be referred to as the shot. There are three common types of projectile.

Bore-calibre Projectiles

Bore-calibre projectiles such as the conventional shell and bullet are of the same diameter as the bore of the gun and so their sides bear directly against the sides of the bore. This type also includes multiple-projectiles in which a number of projectiles are encased within a solid jacket until they have left the gun. Most large bore-calibre projectiles are fitted with a driving band or bands, to allow location, obturation (sealing) and spin of the projectiles within the barrel. To achieve the same functions, projectiles without driving bands rely on the high gas pressures on firing to expand the projectile base and the swaging action of the barrel to crush the projectile to the shape of the rifling.

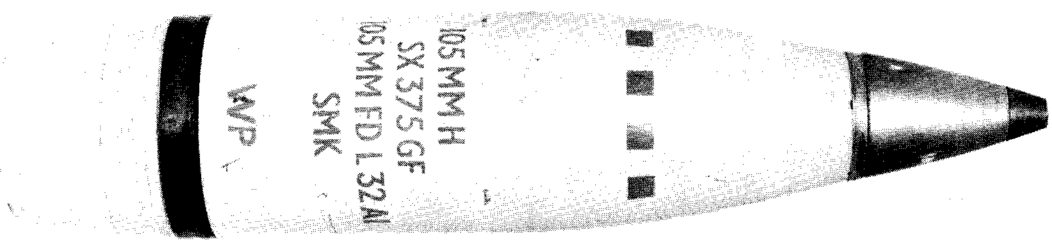


Fig. 2.2 105 mm shell featuring a single driving band

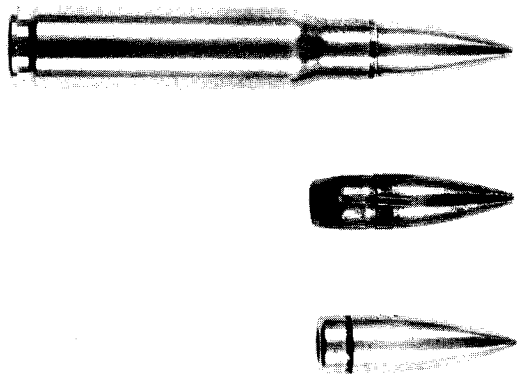


Fig. 2.3 7.62 x 51 mm cartridge together with a fired boattailed bullet and an unfired flat-based bullet

Sabotted Projectiles

There are considerable external ballistic and terminal ballistic advantages to be gained by using sub-calibre projectiles supported by lightweight sabots while in the gun. The sabot will support any shape of projectile, notably including the fin-stabilised projectile, during its motion along the barrel: the sabot is discarded shortly after leaving the gun. There are two basic types of sabot, the axially and the radially discarding sabots. The axially discarding sabot, known as a pot sabot, is pulled off the back of the projectile by the excessive drag generated by its poor aerodynamic shape. Radially discarding sabots consist of a number of petals which are either spun off or blown off by the gas flow after muzzle exit.

A sabot is fitted with a driving band, though when a fin-stabilised projectile is used the driving band is often allowed to rotate freely around the sabot to avoid detrimental spin of the projectile if fired from a rifled gun. Whilst in the gun, the projectile and sabot can together be considered as a single projectile.

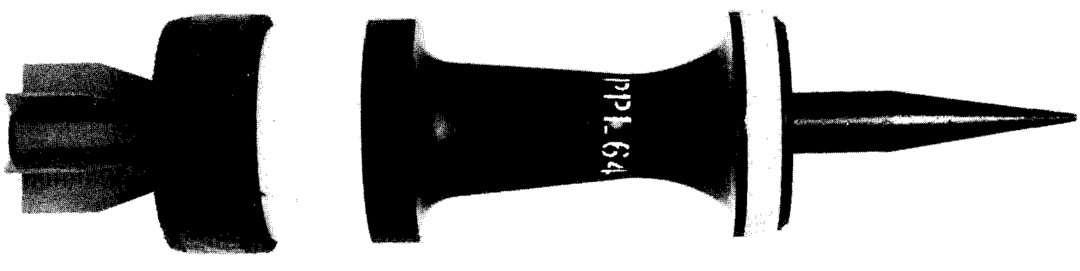


Fig. 2.4 105 mm Calibre APFSDS (Armour Piercing Fin-Stabilised Discarding Sabot) with sabot attached

Shot

The term shot can be applied to loosely supported lead shot (ie, lead spheres) or a number of flechettes positioned ahead of a gas sealing wad, but its use is usually limited to shotguns and warhead fillings. The internal ballistics of guns and shotguns are similar since the behaviour of loose shot driven ahead of a wad is similar to that of other projectiles.

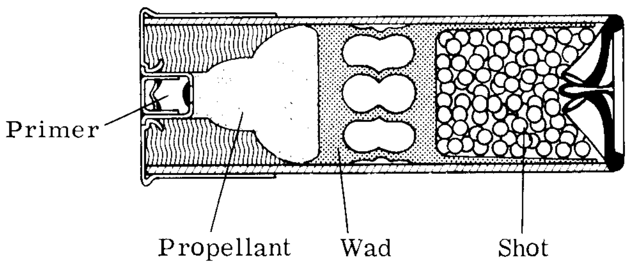


Fig. 2.5 Cross-section of a shotgun cartridge

THE PROPELLANT

Basic Characteristics

Once ignited the components of the propellant, depending on composition, rapidly decompose or react together to produce energetic gases which generate high pressures and temperatures within the gun. The high pressure acts on the base of the projectile and so pushes it along the barrel.

Propellants are available in a vast number of compositions, shapes and sizes. Though some types, especially those used in large calibre gun charges, are formed into long cords, ribbons or slotted tubes, all individual pieces of propellant are referred to here as propellant granules.*

For each gun and projectile combination the propellant type, granule design and quantity used is selected primarily to produce suitable muzzle velocities without exceeding the pressure limitations of the gun. The burning characteristics of a propellant composition are the burning rate constant, the pressure index, the force constant and the co-volume. The burning characteristics of a propellant granule design are the ballistic size and the form function.

*The individual pieces of propellant are often known as grains; to avoid confusion with the grain (a unit of weight = $^1/7000$ pound) sometimes used in ballistics, the propellant pieces are here termed granules.

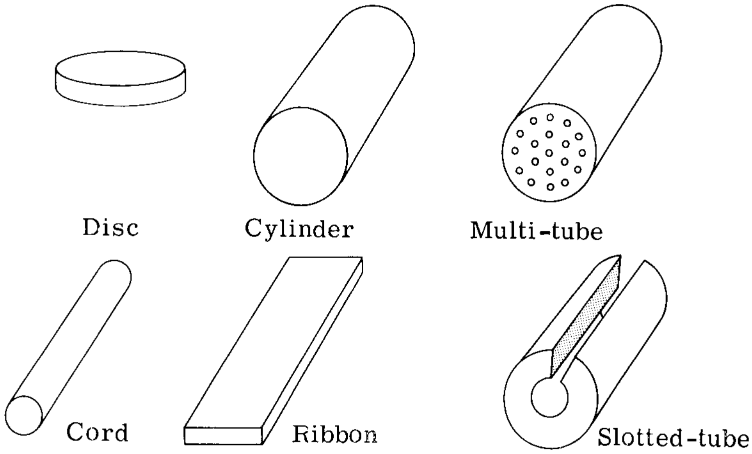


Fig. 2.6 Common shapes of propellant granules

Propellant Compositions

Black powder, often and ambiguously known as gunpowder, is the modern formulation of the earliest propellant; currently it is used in some gun igniters, low velocity guns, and rockets. It consists of a mixture of saltpetre, charcoal and sulphur roughly in proportions of 75:15:10. All other propellants for guns are known as smokeless powders although they are neither completely smokeless nor powders. The three basic types are derived from explosives whose explosive qualities are moderated in the processes which convert them to propellants.

Propellant type	Basic preparation
Single-base	Nitrocellulose dissolved in ether and alcohol.
Double-base	Nitrocellulose dissolved in nitroglycerin.
Triple-base	Nitrocellulose dissolved in nitroglycerin, nitroguanidine added.
Each preparation is then pressed to shape and dried.	

Fig. 2.7 Preparation of smokeless propellants

Double-base propellants are more powerful than single-base propellants, however they suffer from high propellant gas temperatures which can cause excessive barrel erosion and muzzle flash. Triple-base propellants are similarly powerful, but the addition of cool burning nitroguanidine reduces the temperature of the gas to near that of the single-base propellants. Other ingredients added to smokeless propellants are used primarily to control burning rate and suppress decomposition during storage.

Piobert's Law

Before considering what effect the composition and granule design have on the performance of a burning propellant, the mechanical process by which a granule burns must be assumed. It is assumed that on firing, the entire surface of each propellant granule is ignited almost simultaneously and all the surfaces then recede at an identical rate: the granules are said to burn in parallel layers. For example, if the initial granule is cylindrical, it will retain its cylindrical shape throughout the burning process as the diameter and the length of the cylinder will reduce at the same rate. The diameter, being less than the length, will be exhausted first.

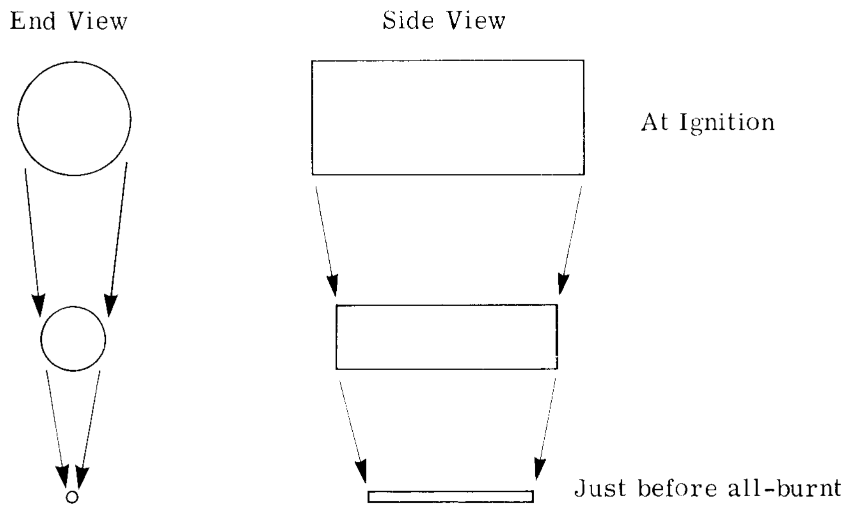


Fig. 2.8 Three stages in the burning of a cylindrical propellant granule

The assumption that propellant granules burn in parallel layers is known as Piobert's Law, and is strongly supported by experimental evidence.

Burning Rate

As a granule of propellant burns, its surface undergoes reactions which convert it to energetic gases. Most of the liberated energy generates high pressures and temperatures of the gas. A small proportion of the liberated energy is conducted into the granule, raising the temperature of successive layers of propellant until they too are ignited. Examples of the ignition temperatures are:

Propellant type	Ignition temperature
Single-base	$315^{\circ}C$ approx.
Double and Triple-base	150°-$170^{\circ}C$

Fig. 2.9 Ignition temperature of smokeless propellants

The ignition process is thus sustained until all of the propellant has been consumed. As burning occurs on all sides, the rate at which a granule reduces in size, known as the burning rate, is equal to twice the rate at which the ignition process spreads through the granule. It is calibrated for a pressure of 1 MPa (approximately 10 atmospheres*): this value is known as the burning rate constant. For example:

Propellant type	Typical burning rate constant
Black powder	18 mm/second/MPa approx.
Smokeless powders	0.5-3 mm/second/MPa

Fig. 2.10 Typical burning rate constants

In some texts, burning rate is taken to be equal to the ignition spread rate, so the burning rate constant appears to be half the expected value. The burning rate constant is affected by changes in the initial temperature of the propellant: an increase in the initial temperature will increase the burning rate.

Pressure Index

The burning rate also increases as the pressure increases, though it does not necessarily increase by the same proportion. The coefficient which relates changes in burning rate to changes in pressure is the pressure index. If the value of the pressure index is 1, the burning rate and pressure rise in direct proportion, so a doubling of the pressure will double the burning rate. The greater the value of the pressure index, the quicker will be the rise in burning rate as pressure increases. For modern propellants the pressure index is usually close to 1 and the burning rate constant is typically 1.5 mm/second/MPa. The increase in burning rate against increasing pressure can be seen in the table below.

Pressure		Burning Rate
MPa	Atmospheres (approx.)	mm/second
0.1	1	0.15
1	10	1.5
50	500	75
380	3800	570

Fig. 2.11 Burning rate against pressure for a typical propellant

From this table it can be seen that a cylindrical stick of typical propellant 1.1 mm in diameter will take over 7 seconds to burn in free air, whereas at the high pressure of 380 MPa, such as may occur in a high velocity gun, the same stick would be completely burnt in less than 2 ms.

Force Constant, Co-volume and Flame Temperature

The amount of energy released by a certain mass of propellant is related to its force constant; thus the powerful double and triple-based propellants have higher

*A pressure of one atmosphere is the natural pressure of the air at sea level.

force constants than the less powerful single-base propellants. The value of the force constant is found by burning a measured mass of propellant in a strong air-tight chamber known as a closed-vessel. The propellant is electrically ignited and the pressure developed within the closed-vessel is measured by a pressure gauge.

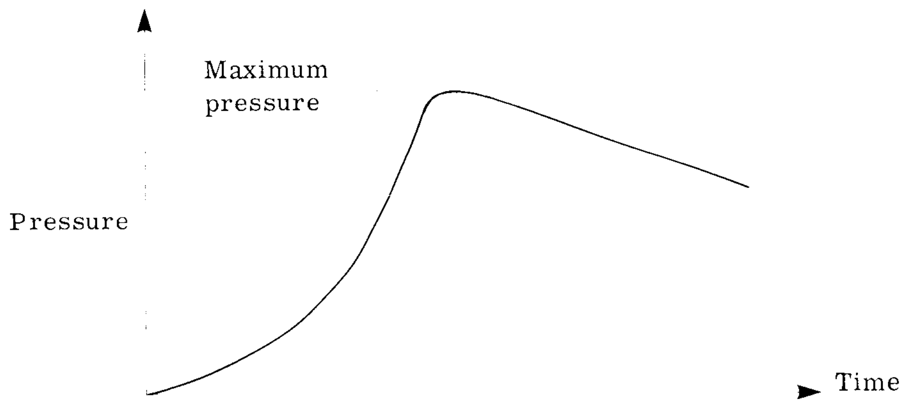


Fig. 2.12 Pressure within a closed-vessel plotted against time

Maximum pressure is reached once all the propellant has burnt; the pressure then begins to fall as the hot gases are cooled by contact with the cold sides of the closed-vessel.

Before the force constant can be calculated, the volume of the closed-vessel must be known. The effective volume of the closed-vessel is less than its actual volume owing to the volume occupied by the propellant molecules. The volume occupied by the molecules of a kilogram of propellant is called its co-volume; this can be determined during a number of closed-vessel firings using a variety of masses of the propellant under test.

$$\text{Effective volume of closed-vessel} = \text{actual volume of closed-vessel} - \text{co-volume of propellant} \times \text{mass of propellant}$$

The force constant can then be calculated:

$$\text{force constant} = \text{maximum pressure} \times \text{effective volume of closed-vessel} \div \text{mass of propellant}$$

Each type of propellant burns with a characteristic flame temperature. This temperature is only effectively achieved in a closed-vessel; in guns the temperature of the gases evolved within the flames is simultaneously cooled by their own expansion. The volume of a closed-vessel is fixed and the rate at which heat is lost by conduction to the vessel sides is slow, so the associated flame temperature is known as the adiabatic* or isochoric** flame temperature.

* Adiabatic = without transference of heat.
**Isochoric = at constant volume.

U.S. standard propellant	Propellant type	Adiabatic flame temperature
M6	Single-base	2570 K (2297°C)
M2	Double-base	3319 K (3046°C)
M30	Triple-base	3040 K (2767°C)

Fig. 2.13 Typical adiabatic (closed-vessel) flame temperatures

In the examples above it is interesting to note that the single-base propellant burns with the lowest flame temperature, and that the triple-base propellant burns at a considerably lower temperature than the double-base propellant even though the two are similarly powerful (ie, have similarly high force constants).

Ballistic Size

When, say, a long cylindrical granule of propellant burns, its length and diameter recede at the same rate in accordance with Piobert's Law of burning. As the diameter is less than the length, it completes burning when the diameter, rather than the length, reduces to zero. From this it can be seen that the diameter is the significant dimension or ballistic size of a long cylinder.

The ballistic size is usually the shortest distance between any two opposing surfaces of a granule. One exception to this is the multi-tube granule which, as will be explained later, continues to burn after the ballistic size has reduced to zero: in this case the significant dimension is called the web size.

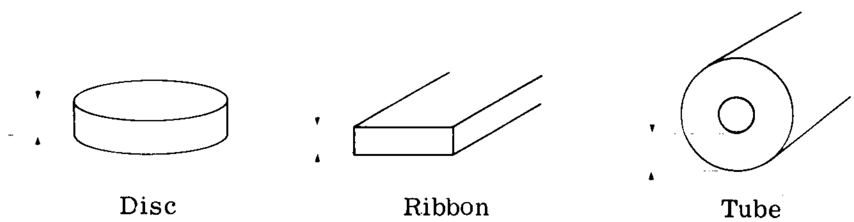


Fig. 2.14 Examples of ballistic size

Form Function

The configuration of the propellant, or form function, as it is known, is important. It is interesting to take the case of numerous long cylindrical granules of propellant. At ignition they possess a relatively large total surface area. The rate at which propellant gas is produced is directly proportional to the total surface area of all the granules so there is a relatively fast initial production of gas. As the burning proceeds, the surface area reduces and results in a corresponding reduction in the relative rate at which gas can be produced. Each granule is

simultaneously being subject to variations in pressure which affect the burning rate and so also affect the rate at which gas is produced. Finally, the surface area of the propellant granules falls to zero and the production of gas stops. Such a granule shape, in which the surface area diminishes as the burning progresses, is said to be degressive.

For each granule shape, a geometric relationship exists between the fraction of propellant burnt and the fraction of the ballistic size remaining at any moment. This relationship is called the form function. The degressiveness of a granule is described mathematically by the form function coefficient that appears in the form function. Degressive granules, such as long cylinders, have positive coefficients. By comparison, a thin disc shaped granule maintains almost the same surface area throughout the burning process, and so has a coefficient very close to zero.

Tubular granules are frequently used as they are less degressive than cylindrical granules. When long tubes are used, they are slotted along one side to avoid excessive pressures within the tube that would otherwise cause the tubes to disintegrate.

Some propellant granules, on the other hand, are designed to generate most gas near the end of the burning process. This is achieved by either forming the propellant into tubes, or by doping the degressive outer layers of the granules with burn suppressants or ignition inhibitors. Such granules have negative form function coefficients and are said to be progressive. The initial production of gas is slowed by suppressants in the outer layers of solid suppressed burn granules. Once the suppressed layers have burned, the burning proceeds at the normal degressive rate.

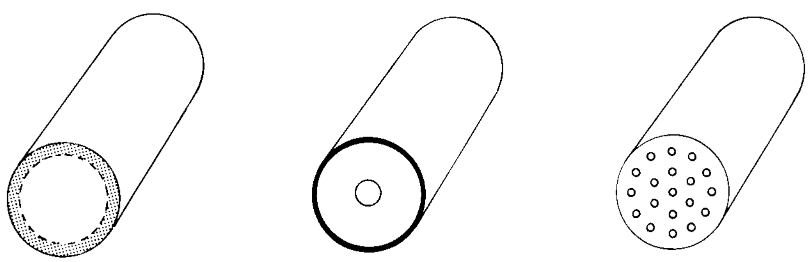
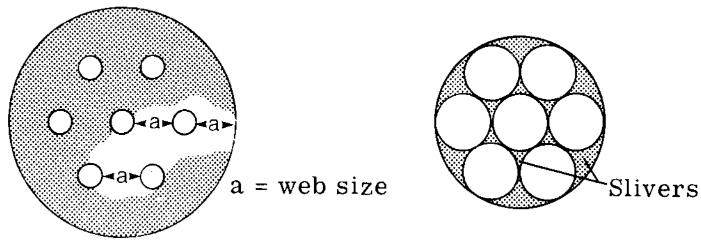


Fig. 2.15 Types of progressive granules

Whilst the outer surface of a tube burns degressively, its progressive inner surface increases in area as it burns: but if the outer surface is completely inhibited then the granule will burn progressively.

Alternatively, the degressive outer surface can be compensated by the progressive burning of a number of holes through the granule; this is the principle applied to multi-tube granules to produce neutral or slightly progressive burning. Once the

web size of a multi-tube granule has reduced to zero, it breaks into so-called slivers which continue to burn until all the propellant has been exhausted.



Dimensions at ignition　　Formation of slivers

Fig. 2.16 Cross-sections of 7 hole multi-tube granule

Granule shape	Approximate form function coefficient	
Random chips	positive	Most degressive
Spherical	positive	
Cylindrical/Cord	positive	
Disc	positive, near zero	
Tube/Slotted tube	positive, near zero	
Ribbon	positive, near zero	
Solid, suppressed burn of outer layers	near zero	Neutral
Multi-tube	near zero	
Tube, inhibited ignition of outer surface	negative	Most degressive

Fig. 2.17 Examples of granule shapes and their form function coefficients

THE PRIMER

A primer initiates the burning of a propellant by releasing hot gases and particles into the charge. The surface temperature of granules in the flow of hot primer gases rises until they are ignited. The flow of hot primer and propellant gases then spreads throughout the chamber perpetuating the flame spread like a wave.

The wave form flame spread is accompanied by a pressure wave generated as a result of the gas flow and enhanced production of propellant gases in regions of high relative pressure. The pressure waves are reflected at the chamber sides

and so pass back and forth through the propellant. This behaviour is normally detected as rapid fluctuations in the pressure measured at the base and projectile ends of the chamber. The oscillations fade once the projectile begins to accelerate along the bore. Such oscillations can lead to inconsistent burning and excessive chamber pressures: this serious problem is frequently encountered with modern guns using very slow burning propellants in relatively large chambers.

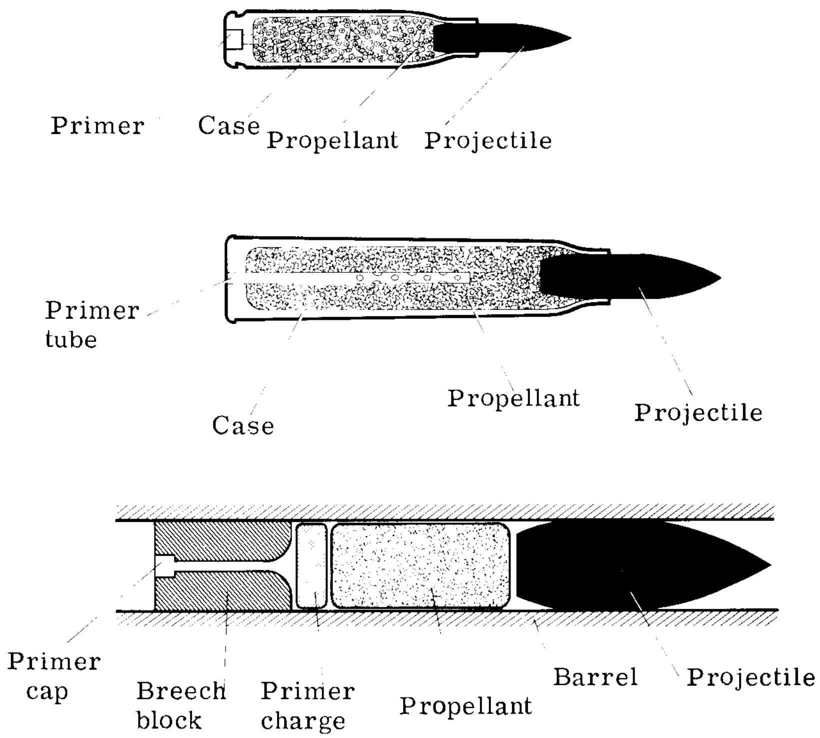


Fig. 2.18 Configuration of igniter and propellant charge in typical ammunition

The importance of the primer and propellant charge interaction cannot be overstressed. Any inconsistencies in ignition will manifest themselves on the entire ballistic sequence: consequential variations in muzzle velocity and recoil effects will reduce accuracy.

THE FIRING SEQUENCE

The firing sequence is usually initiated by the ignition of the primer. This causes the primer charge combustion products, consisting of hot gases and incandescent particles, to be injected into the propellant. The hot gases flowing between the propellant granules ignite the granule surfaces; the primer and propellant combustion products then act together, perpetuating the flame spread until all the propellant granules are ignited.

At first the chamber is virtually sealed by the projectile, so the gases and energy liberated by the primer and propellant cannot escape: there is a resultant dramatic increase in the pressure and temperature within the chamber. The burning rate of the propellant is roughly proportional to the pressure, so the increase in pressure is accompanied by an increase in the rate at which further gas is produced. Without any means to check the accelerating production of gas, the gun would explode.

The rising pressure is moderated by the motion of the projectile along the barrel. The pressure at which this motion begins is the shot-start pressure. The projectile will then almost immediately encounter the rifling and the projectile will slow or stop again until the pressure has increased sufficiently to push it into the constricting bore. The driving bands or the surface of the projectile itself, depending on design, will be engraved to the shape of the rifling if the bore is rifled. The resistance then falls, allowing the rapidly increasing pressure to accelerate the projectile.

As the projectile moves forward it leaves behind it an increasing volume to be filled by the high pressure propellant gases. The propellant is still burning, producing high pressure gas so rapidly that the motion of the projectile cannot fully compensate: as a result, the pressure continues to rise until the peak pressure is reached. The peak pressure is attained when the projectile has travelled about one tenth of the total length of a full length gun barrel.

The extra space being created behind the rapidly accelerating projectile then exceeds the rate at which high pressure gas is being produced, and so the pressure begins to fall. The next stage is the all-burnt position at which the burning of the propellant is completed. However, there is still a considerable pressure in the gun so, for the remaining motion along the bore, the projectile continues to accelerate: as it approaches the muzzle the propellant gases expand, the pressure falls, and so the acceleration lessens. At the moment the projectile leaves the gun the pressure will have reduced to about one sixth of the peak pressure.

The flow of gases following the projectile out of the muzzle provides additional acceleration for a short distance so that the full muzzle velocity is not reached until the projectile is some distance beyond the muzzle. After this, the projectile is soon lost from the gun's influence and begins its fast, independent flight.

This entire sequence, from primer firing to muzzle exit, typically occurs within 15 milliseconds. For a handgun the sequence may take less than 1 millisecond, but perhaps as much as 25 milliseconds for a large artillery gun.

Time and Space Curves

Pressure gauges fixed in the side of the chamber can measure the propellant gas pressure during the firing sequence, whilst velocity and displacement transducers can measure the velocity and travel of the projectile along the bore. The measurements taken can be automatically recorded against time, so the plotted data produces graphs which are known as the pressure/time, velocity/time and travel/time curves and are illustrated in Fig. 2.19.

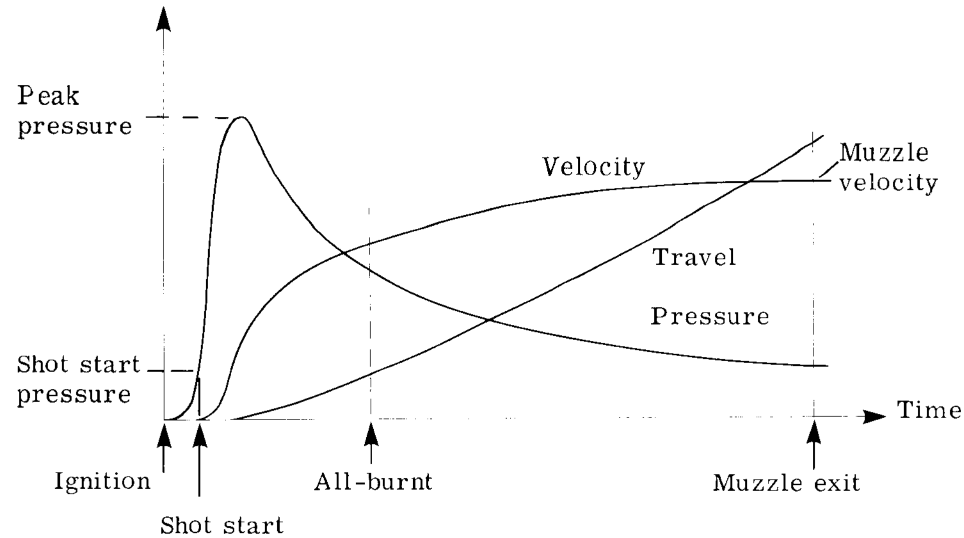


Fig. 2.19 Typical pressure/time, velocity/time and travel/time curves

The pressure and velocity data can also be plotted against the simultaneous displacement of the projectile; these graphs are known as the pressure/space and velocity/space curves.

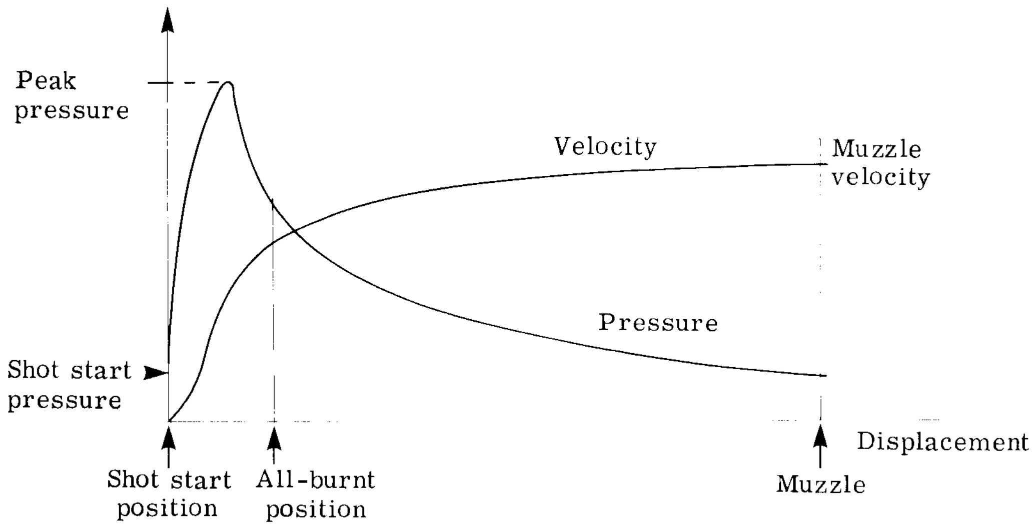


Fig. 2.20 Typical pressure/space and velocity/space curves

Forces Acting on the Projectile

There are two opposing forces acting on a projectile within the gun: the propelling force is due to the high pressure propellant gases pushing on the base of the

projectile, whilst the frictional force between the projectile and bore, which includes the high resistance during the engraving process, opposes the motion of the projectile. Additionally, the reaction between the projectile and the rifling of a rifled gun translates a small part of the propelling force into a torque which causes the projectile to rotate. So we can derive the following equations:

Propelling force = Propellant gas pressure x Area of projectile base.

Ignoring the small energy translated into the rotation of the projectile within a rifled gun:

Total force on projectile = Propelling force - Frictional force.

If the gun is elevated, the frictional force will include a slight additional force due to the weight of the projectile. If the total force and projectile mass are known, it is possible to calculate the acceleration of the projectile:

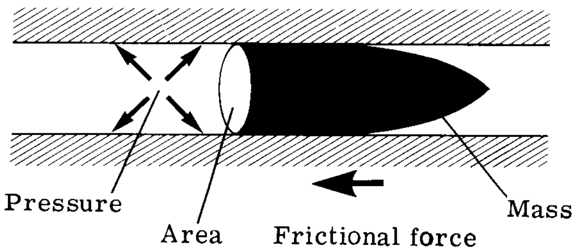


Fig. 2.21 The four factors which determine projectile acceleration

The pressure and frictional force vary continuously during the firing sequence, and so the acceleration varies also.

In practice, the projectile mass and base area will be known, the gas pressure can be measured and recorded during the firing sequence, and the acceleration deduced from the observed motion of the projectile. Consequently it is possible to calculate the frictional force resisting the motion of the projectile.

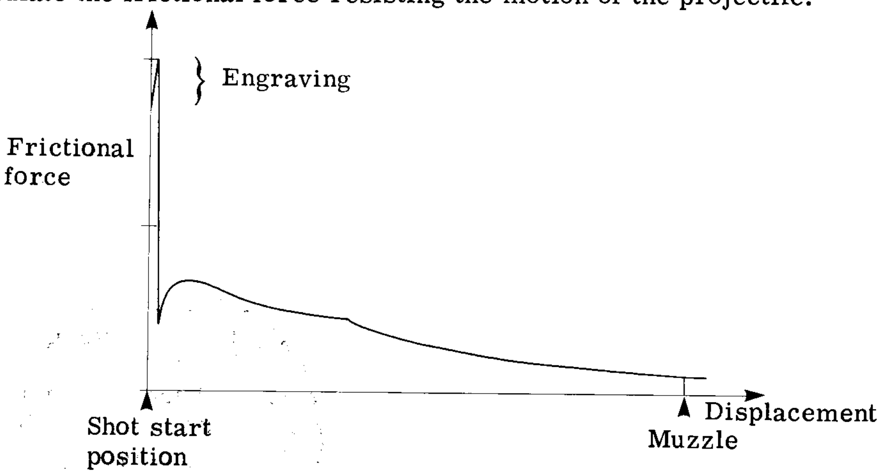


Fig. 2.22 Typical frictional force during firing

Distribution of Energy

When a propellant burns it releases a large amount of energy in the form of hot propellant gases. As the firing sequence progresses, a large proportion of this energy is converted into other forms. The approximate distribution of energy at the end of the firing sequence is given in Fig. 2.23.

Motion of projectile	32%
Motion of propellant gases	3%
Frictional losses	3%
Heat loss to gun and projectile	20%
Heat retained by propellant gases	42%
Total energy liberated by propellant	100%

Fig. 2.23 The approximate distribution of liberated energy

This table reflects the basic function of the gun, which is to convert the heat of propellant gases into useful kinetic energy of the projectile. In this case, the gun has achieved its function with 32% efficiency on the basis of:

$$\text{Percentage propulsive efficiency} = \frac{\text{Muzzle energy of projectile}}{\text{Total heat of propellant}} \times 100$$

Unless the propulsive efficiency varies from shot to shot it is of no significance to the gun's efficiency to deliver a projectile to a specified target.

There are some extra minor effects on the efficiency of a gun. For example, if the gun is rifled, the projectile will exit the gun with a rotational energy which, for a medium calibre, will amount to only 0.15% of the total energy liberated by the propellant. Then some energy is lost to the recoil motion of the gun; this will account for between 0.02% and 0.5% of the total available energy. In simple ballistic studies recoil energy is negligibly small: to illustrate this, some typical recoil energies, given as a percentage of total available energy in each case, are:

Weapon	Relative recoil energy
5.56 mm Rifle	0.1%
120 mm Tank gun	0.2%
.44 Magnum pistol	0.5%

Fig. 2.24 Approximate recoil energies relative to total available energy

An inefficient gun/ammunition design may fail to reach all-burnt. In this case unburnt propellant will be ejected through the muzzle: this unburnt propellant can be a significant cause of energy loss, but should not normally happen.

Pressure of Propellant Gases

There are many complex factors contributing to the pressure at any moment within the gun. However, as a simple guide, the propellant gas pressure is roughly proportional to heat retained by propellant gases ÷ volume of propellant gases. Shortly after ignition, large pressure fluctuations may occur owing to oscillations of the gas within the chamber. Later, when the projectile is moving rapidly, there will be a fall in pressure towards the base of the projectile as friction between the propellant gases and the bore resists the motion of the gases along the bore.

Peak Pressure

There are two main factors which contribute to a high peak pressure. The first is the rapid liberation of energetic gases during the early stages of the firing sequence and the second is the high projectile mass.

A rapid liberation of energy requires either a large total surface area of the propellant granules, a high value of force constant or a fast burning rate. The size, shape and number of granules will determine the total surface area, whilst the force constant and burning rate are dependent on the composition of the propellant.

A projectile of high mass will tend to resist acceleration, so restricting the volume into which the gases can expand and so increases the peak pressure. Conversely, if the early rate of energy liberation or the projectile mass are reduced, the peak pressure will be less.

Gun/ammunition	Typical peak pressure
5.56 x 45 mm (Rem. 223)	354 MPa
120 mm APDS	425 MPa

Fig. 2.25 Examples of peak pressure

When progressive granules are used, the initial liberation of energy is limited, thereby limiting the peak pressure to the safe limits of pressure and erosion that may be tolerated by the gun.

All-Burnt

All-burnt refers to the moment at which all of the propellant has been burned; this happens almost simultaneously for all the granules though the exact instant for each granule will be dependent on its local conditions within the gun during the firing sequence.

The position of the projectile at all-burnt is largely dependent on the peak pressure and the form function of the propellant granules. A high peak pressure implies that the pressure is relatively high throughout the firing sequence and, as

the burning rate is roughly proportional to pressure, all-burnt will be reached very rapidly. Similarly, a low peak pressure implies a late all-burnt.

Progressive propellant charges, which are discussed more fully later in the chapter, usually produce late all-burnt positions due to their moderated peak pressure. Indeed, when multi-tube granules are used, all-burnt is frequently never reached and some of the remaining slivers of propellant are ejected in an intense muzzle flash. After peak pressure has occurred, the fall in pressure, and consequent burning rate, are dependent on the form function. Progressive granules tend to sustain the pressure slightly so that at all-burnt the pressure is higher than with degressive granules.

If all-burnt occurs early, when the projectile has travelled only a short distance, the results will be an increased propulsive efficiency, a reduced muzzle blast and flash and an increased consistency of muzzle velocity from shot to shot.

After all-burnt, all the available propellant gases are able to contribute to the propulsion of the projectile for its remaining travel along the bore. If all-burnt occurs early, all the available gases will be able to act on the projectile over an increased distance. The greater the distance travelled by the projectile after all-burnt, the greater the expansion of the propellant gases. Thus the propulsive efficiency is increased and, for a given projectile mass, the muzzle velocity is increased. As expansion of the gases reduces their pressure and temperature, an early all-burnt will also result in reduced blast and flash at the muzzle. The use of a longer barrel will have a similar effect to an early all-burnt position unless the barrel is excessively long, in which case the sustained projectile/bore friction will exceed the propulsive force and reduce the muzzle velocity.

In practice the greatest variation of projectile velocities observed from shot to shot occurs at the position of all-burnt; but the variation of velocities lessens towards the muzzle, so the longer that projectiles travel after all-burnt the less the variation of muzzle velocities, which again is an argument for having an early all-burnt position.

Progressive Propellant Granules

Gun systems usually use the largest propellant charge that may be inserted into the chamber to achieve the highest practical peak pressure that the gun can withstand. The question then arises, how is it possible to increase the muzzle velocity if the projectile mass remains the same?

The burning rate, form function, ballistic size and number of granules can be adjusted to give an earlier all-burnt; the resultant increase in propulsive efficiency will probably be small, and the peak pressure may exceed the limitations of the gun.

A larger increase in the muzzle velocity implies that the amount of energy supplied by the propellant must be increased, which can only be achieved by using a propellant with a higher force constant. Providing the gun can withstand the very high pressures developed, the muzzle velocity will be greatly improved. However, the

peak pressure can be moderated by forming the propellant into neutral or progressive granules rather than the more usual degressive granules. The majority of the energy will then be liberated after the peak pressure has occurred, so sustaining the pressure at high levels until all-burnt is reached. This delayed liberation of energy will result in a reduced propulsive efficiency; nonetheless, the losses due to inefficiency will be small compared to the large increase in available energy. The low efficiency will be reflected by an increase in muzzle blast and flash. The delayed liberation of energy will also tend to give inconsistent muzzle velocities: in consequence progressive propellant granules are usually limited to guns where high muzzle velocity is more important than accuracy or low blast and flash.

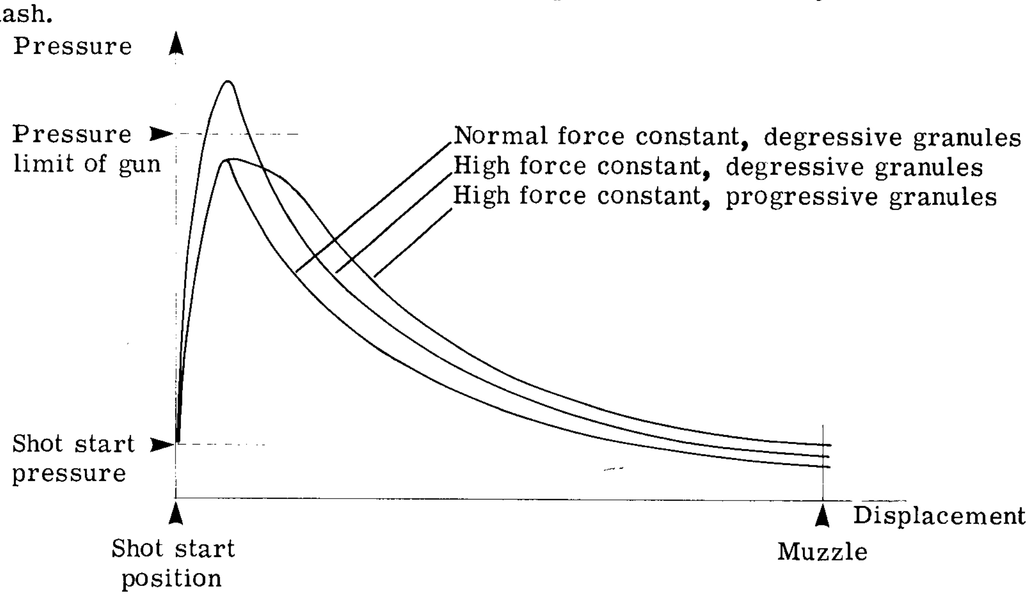


Fig. 2.26 Pressure/space curves for three full charges of propellant

BARREL LIFE

The life of an in-service gun, usually stated in terms of the number of times it can be fired accurately, is limited by the three prime causes of internal barrel wear, which are corrosion, abrasion and erosion.

Erosion is the most significant as is explained later.

Corrosion

Corrosion is normally the most insignificant limit to barrel life as modern propellant combustion products are chemically non-corrosive. However, gunpowder and other primer compounds may cause corrosion; and as most barrels are made of steel, they may rust in damp climates. Such causes of corrosion can easily be avoided by suitable treatment of the bore after use, and by using self-cleaning and lubricating projectiles.

Abrasion

The friction of the projectile against the bore will cause some wear of the bore. The extent of this abrasion varies with the type of projectile used, though in most cases the amount of purely abrasive wear is relatively slight. In large calibre guns the use of plastic driving bands reduces abrasion, but small calibre guns, in which separate driving bands are not used, rely on the choice of projectile surface material and projectile lubrication to control abrasion. The choice of projectile surface material is not simple; indeed, hard armour-piercing bullets have on occasion produced less wear than did standard bullets.

In some circumstances, foreign material such as sand may repeatedly enter the bore between shots; the result would be a serious shortening of the gun's life. Large quantities of material obstructing the bore would cause local expansion of the barrel as the projectile passes, possibly resulting in the muzzle end of the gun being blown off.

Conversely, the abrasion may wear the projectile rather than the bore: the accumulation of material lost from a number of projectiles causes so-called leading of the bore, and the gun will consequently lose accuracy. Leading can be removed by scrubbing of the bore, so it has no effect on the life of the gun. Also, lubrication of the projectile can often be used to suppress both leading and abrasion of the bore.

Erosion

Erosion, the major cause of barrel wear, is caused by the transfer of heat from the energetic propellant gases to the bore. The region of greatest erosion is near the start of the rifling, or at the same relative position in a smooth-bore gun. The collision of energetic gases against the bore surface produces rapid local heating. The hot steel reacts with the propellant gases to produce a weak layer of brittle compounds that is removed during subsequent firings by the abrasive action of the propellant gases, propellant granules, and projectiles. The removed material is carried out with the propellant gases through the muzzle.

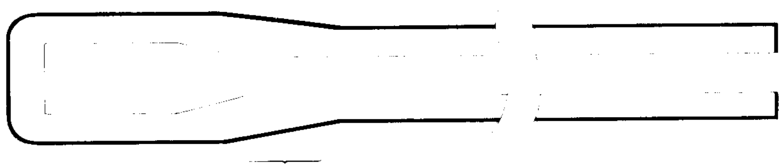


Fig. 2.27 Erosion of the gun

Although the temperature of the entire barrel rises slightly as a result of firing, it is the high temperature of the bore surface during the firing sequence that determines the rate of wear. The higher the bore surface temperature during firing, the greater the rate of wear.

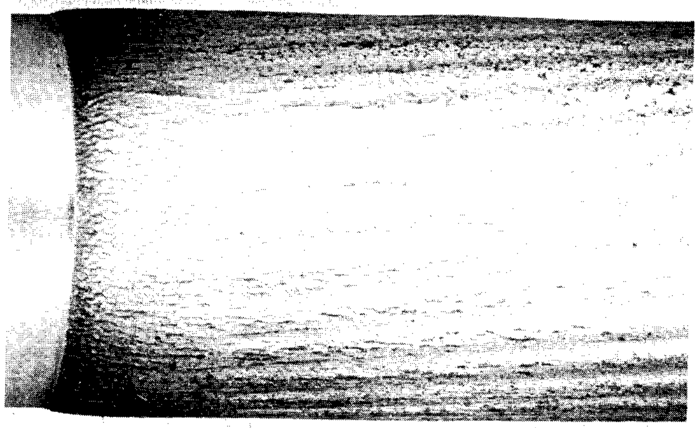


Fig. 2.28 An eroded gun sectioned to show enlargement of the bore and worn rifling

The rate of erosion is dependent on seven factors:

1. propellant
2. wear additives
3. rate of fire
4. barrel cooling
5. calibre
6. barrel material
7. gun design.

The primary features of a propellant charge which control the rate of erosion are the force constant and form function. A high force constant implies high temperatures of the propellant gas and a consequent high rate of erosion. The erosion rate is very sensitive to even small changes in gas temperature, so it is important to avoid excessively high force constants. However, relatively high force constants may be used if the peak pressures are moderated by the use of progressive granules, because the very high rates of heat transfer that accompany very high pressures are avoided. Low burning rates and pressure indices also reduce erosion, but at the cost of reduced muzzle velocity.

The rate of erosion can be reduced by the addition of small quantities of chemically inert wear additives to the propellant. On firing, the wear additives will tend to settle on the bore surface; the insulative layer formed will resist heat transfer from the propellant gas to the barrel, thus reducing the rate of erosion. Successful additives include titanium dioxide/wax and talc/wax mixtures.

During rapid fire, when there is little chance for cooling between shots, the overall temperature of the barrel will rise, and so the bore surface temperature during firing will also rise. The consequence is a shortened barrel life unless a cooling

system is used. Coolant can be pumped through tubes in the barrel and then passed through ventilated radiators. Aircraft guns simply use forced airflow during flight to maintain cooling.

Erosion is most severe in guns in which the circumference of the bore is large compared with the cross-sectional area of the bore (see Fig. 2.29). This describes small arms and the relatively small large calibre guns. This problem is conveniently overcome when discarding sabot projectiles are used.

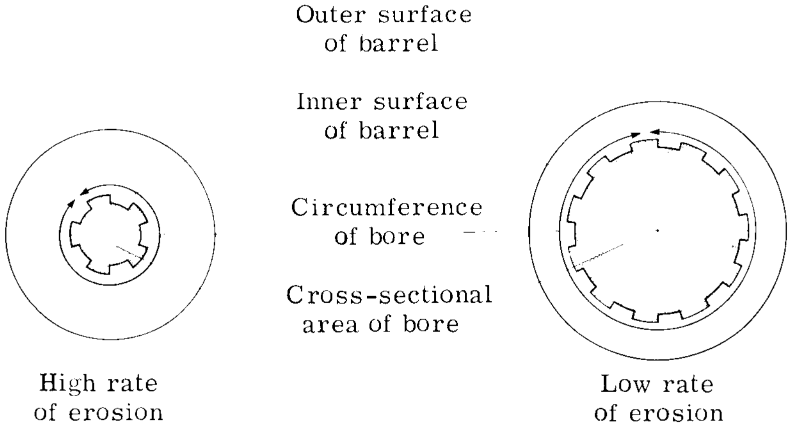


Fig. 2.29 Cross-section of the bore and relative rates of erosion

Barrel steels containing alloys, with such materials as tungsten and chromium, do much to increase the erosion resistance, and chromium electroplating of the bore can almost completely eliminate wear by erosion. However, they are expensive and difficult to produce.

Erosion can, however, be further minimised by careful design of the gun. Probertised rifling is an excellent example of this. As erosion is accentuated by rifling, the gun is made smooth-bore at the shot-start position where erosion is greatest. The depth of rifling increases towards the muzzle, cutting into the driving bands during its motion along the barrel. Then the bore narrows and the rifling decreases again near the end of the bore, squashing the driving bands flat to improve the projectile's flight characteristics.

Probertised rifling is in fact smooth-bored at the muzzle. The result is that the life of a Probertised gun is up to 6 times longer than that of a similar conventional gun, though both the gun and its ammunition are expensive to produce.

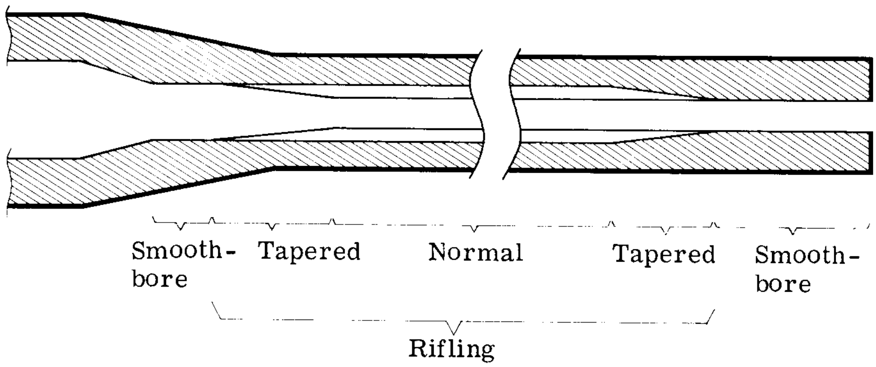


Fig. 2.30 Rifling depth in a Probertised barrel

Practical Barrel Life

A barrel can be considered worn out when the gun cannot achieve its required accuracy in normal use. The point at which this occurs varies, depending on the type of gun, ammunition used, and minimum acceptable accuracy. The amount of wear is measured at the commencement of rifling, and is the percentage increase in diameter of the bore. Whilst barrel life is defined as the number of shots required to increase the bore diameter by 5%, the practical increase in diameter that can be tolerated is usually between 1% and 5%.

The muzzle velocity of a gun is determined by factors which also affect its rate of wear: so it is possible to relate barrel life to muzzle velocity, though the relationship is not simple. Generally, a small increase in muzzle velocity will considerably shorten the life of a gun.

Most guns that suffer short barrel life do so as a result of high rates of erosion caused by high muzzle velocities or rapid rates of fire. Such guns include anti-tank guns, anti-aircraft guns, high velocity rifles and machine guns.

Gun/ammunition	Approx. muzzle velocity	Typical practical barrel life
.22 (5.5 mm) LR	320 m/s	1,000,000 rounds
5.56 x 45 mm (Rem. 223)	990 m/s	10,000 rounds
88 mm anti-aircraft	1000 m/s	100 rounds
120 mm anti-tank (sabotted)	1370 m/s	100 rounds
.50 inch Browning, Rapid Fire	890 m/s	500 rounds

Fig. 2.31 Examples of practical barrel life

Note that the .50 inch Browning machine gun firing at 1000 rounds per minute has a barrel life of just 30 seconds.

Barrel Distortion

A secondary effect, due to the transfer of heat to and from the barrel, is barrel distortion. Heating by the propellant gas, and uneven cooling by, say, wind will cause the barrel to bend slightly, so affecting the accuracy. This can be a serious problem for some modern thin-barrelled guns. Fortunately, it can be controlled by the choice of propellant and wear additives, insulation of the barrel by lagging on the external surfaces and, where necessary, an adequate cooling system.

Fig. 2.32 Chieftain tank fitted with 120 mm lagged gun

RECOILLESS GUNS

A recoilless gun consists of a tube open at both ends. The projectile is propelled through a conventional muzzle, and the propellant gases are ejected through the open breech of the gun. The forward momentum imparted to the projectile equals the backward momentum imparted to the propellant gases; consequently the gun does not move, and it is therefore recoilless. In some alternative recoilless systems, the backward momentum is provided by the ejection of a counter-mass from the rear of the tube. In its simplest form, the recoilless gun is no more than a rocket launcher tube, but the term is usually applied to the more efficient jet reaction gun. (See Fig. 2.33).

During the firing sequence, the pressure within the recoilless gun increases, thus efficiently driving the projectile forward. Simultaneously, the pressure acts on the breech, tending to drive the gun backwards. This force is balanced by a high velocity jet of propellant gases ejected through a nozzle, or nozzles, which generates an equal forward reaction force.

The advantages of the recoilless gun are that it is lightweight, of simple construction and has a long barrel life. Its disadvantages are that it requires a charge some 3 to 4 times greater than that of a conventional gun, it has a low maximum achievable muzzle velocity and its accuracy is poor.

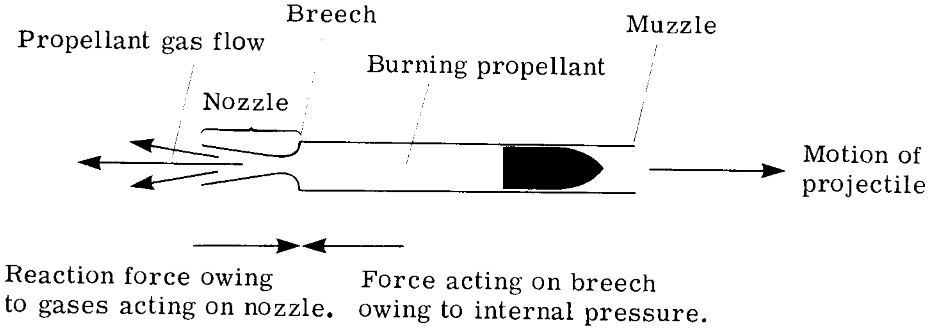


Fig. 2.33 The principle of the recoilless jet reaction gun

Fig. 2.34 LAW 80 (early model) anti-tank recoilless rocket launcher

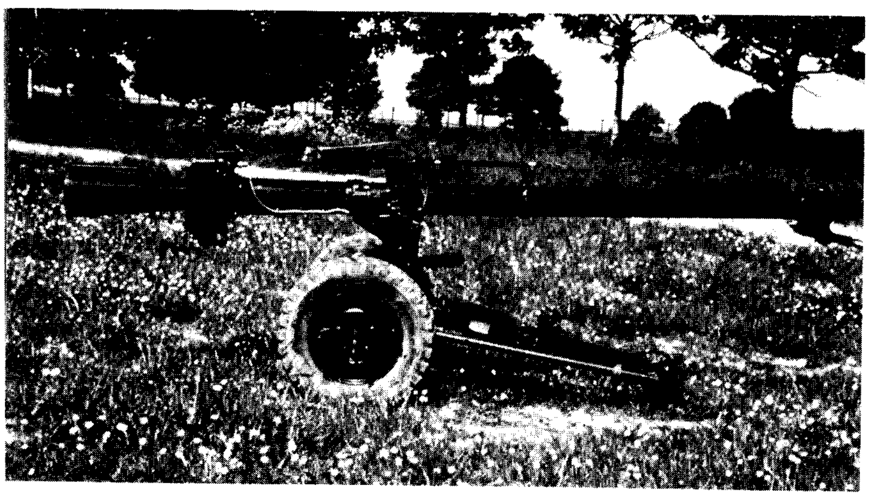


Fig. 2.35 120 mm Conbat recoilless jet reaction gun

So far in this first section on internal ballistics, we have looked, in a descriptive way, at the various factors which affect internal ballistics. Some readers, who do not have a mathematical background, will find it sufficient. Others, with a mathematical turn of mind, will be interested in the development of the theme in a more theoretical way in Section II.

SELF TEST QUESTIONS

QUESTION 1 How does the combustion of propellants differ from the combustion of conventional fuels?

Answer ....

....

QUESTION 2 What are the purposes of driving bands?

Answer ....

....

QUESTION 3 Which burning characteristics are dependent on the composition of a propellant, and which are dependent on the shape of the granules?

Answer ....

....

....

QUESTION 4 What is the order of events of a typical firing sequence?

Answer ....

....

QUESTION 5 What are the characteristics upon firing of guns using multi-tube propellant granules?

Answer ....

....

....

QUESTION 6 Which configuration of chamber size and propellant burning rate promotes pressure waves in the chamber?

Answer ....

....

QUESTION 7 In what fashion does the surface area of a progressive granule of propellant change during burning, providing that it is not treated with burn suppressants or inhibitors?

Answer ....

QUESTION 8 What are the arguments for and against barrel lagging?

Answer ....

....

....

QUESTION 9 In the case of a typical spin-stabilised projectile, what proportion of its kinetic energy at muzzle exit is due to rotation?

Answer ....

....

....

QUESTION 10 Why is the temperature of the propellant gases within a gun lower than the flame temperature?

Answer ....

....

....

ANSWERS ON PAGE 201

Internal Ballistics — Part II

SCOPE

To produce a mathematical model of the internal ballistics of the gun as a whole, it is necessary to model the dynamics of each of the components. The treatment given here is a simplification of both the real gun and typical models of the gun as only the main internal ballistic components are considered. These components are the propellant, the propellant gases and the projectile. In the following pages five equations will be derived that model the burning of the propellant, the behaviour of the gases and the resultant motion of the projectile. In deriving these equations, well known principles of physics will be applied, together with explanation of many of the processes particular to the gun. Although referred to, the equations governing some of the more complex processes are not given; these can be found in various reference books which deal with internal ballistics more fully. Both classical and contemporary applications of the five equations are discussed, and an outline is given of the two main types of computer gun simulation.

BURNING RATE LAW

Looking first at an individual granule or propellant, we need some idea of the way in which it burns. In obeyance with Piobert's law of burning the entire surface of a burning propellant granule regresses at the same rate, and so recedes in parallel layers. As a granule burns from all sides, the rate at which it reduces in size is twice the rate of regression of just one side. This rate of reduction in size, known as the burning rate*, varies with the pressure of the surrounding gases in accordance with the following equation:

$$\text{burning rate, } \frac{ds}{dt} = BP^a$$

in which s is the reduction in size of the granule;

*In some references, notably American, burning rate is taken to be equal to the rate of regression, and so the burning rate constant appears to be half the expected value.

t, time in seconds;

P, gas pressure in the region of the granule;

a, the pressure index of the propellant composition

and B is the burning rate constant of the propellant composition.

The ballistic size, the initial size, of the granule is represented by D. Now, if at some instant after ignition when the size of the granule has reduced by a distance s, and the remaining size of the granule is a fraction f of the original size D, then:

$$D - s = fD.$$ D

At ignition f equals one, decreases in value as the burning proceeds and equals zero when all burnt is reached. This relationship between s and f, differentiated with respect to time gives

$$-\frac{ds}{dt} = D\frac{df}{dt};$$

and by substitution for $\frac{ds}{dt}$ in the burning rate equation leads to:

$$\frac{df}{dt} = -\frac{B}{D}P^a.$$

This function can now be integrated with respect to time, so the fraction f of the ballistic size burnt at time t can be equated with the variations of pressure during firing:

$$f = -\frac{B}{D}\int_0^t P^a \, dt. \qquad \text{(Equation 1)}$$

The value of $\frac{B}{D}$ is known as the propellant vivacity. For a propellant of given pressure index and granule shape, it is possible to vary the values of B and D without affecting the internal ballistics of the gun in which it is used, providing that the vivacity remains the same. Of course, the extremes of ballistic size would cause problems in practice.

FORM FUNCTION

We next need to relate the proportion of propellant burnt with the remaining fraction f of the ballistic size. The relationship is purely geometrical, and depends only on the shape of the granules used. Writing z as the fraction of propellant volume burnt at some instant t, then

$$z = \frac{\text{original volume - volume at time t}}{\text{original volume}}$$

$$z = 1 - \frac{\text{volume at time t}}{\text{original volume}}$$

Now, if the granules have a cross-sectional area A_g and a length L then

$$z = 1 - \frac{A_g L \text{ (at time t)}}{A_g L \text{ (original)}}$$

As the length L of the granules is generally much greater than the ballistic size D, the relative change in granule length during burning is negligibly small. It is therefore possible to make the approximation:

$$L \text{ (at time t)} = L \text{ (original), and so}$$

$$z = 1 - \frac{A_g \text{ (at time t)}}{A_g \text{ (original)}}$$

Taking a long cylindrical granule as an example,

$$A_g \text{ (original)} = \frac{\pi D^2}{4}$$

and

$$A_g \text{ (at time t)} = \frac{\pi (fD)^2}{4}$$

Putting these into the equation for z gives

$$z = 1 - f^2,$$

and this can be expanded to give:

$$z = (1 - f)(1 + kf). \qquad \text{(Equation 2)}$$

This equation relating z and f is known as the form function and here, in the case of the long cylindrical granule, the form function coefficient k equals one. Different values of k allow the form function to be used for a variety of propellant shapes. For example, for long tube or slotted tube, $k = 0$; and for long ribbon of thickness D and width wD, $k = \frac{1}{w}$. In the case of multitube granules, such as the 7 hole cylinder, an empirical form function is used: the ballistic size is taken to be 1.28 times the web size, and $k = 0.12$.

THE CLOSED-VESSEL

As was seen in part 1, the closed-vessel is the primary tool in the calibration of propellants. By extending the principles governing the behaviour of propellant gases in the restricted conditions of a closed-vessel, it is possible to grasp an understanding of the complex gas behaviour during the firing sequence of a gun.

Equation of State

The starting point for the underlying theory is Van der Waal's equation of state for an ideal gas:

$$PV = nR_uT$$

where P is gas pressure,

V is gas volume,

n is the number of moles* of gas,

R_u, the universal gas constant

and T is the absolute gas temperature.

This equation describes the relationship between P, V, n and T for a gas consisting of very simple molecules. However the real gases evolved by a burning propellant are relatively complicated and so do not behave exactly as Van der Waal's equation predicts. The most significant cause of discrepancy between theory and practice is the coalescence of gas molecules as they are forced together at the high pressures achieved. This effect is known as condensation as it is, in effect, similar to the condensation of a mist of water droplets from humid air. As a result, the volume of the gas is reduced by the condensation of apparently non-gaseous particles from the high pressure gases. To compensate for the reduction in volume, the equation of state is assumed to be of the form

$$P(V - mc) = nR_uT$$

where m is the mass of propellant burnt

and c is the co-volume of the propellant.

This, the Noble-Abel equation, is used as the equation of state for the gases of both the closed-vessel and the internal ballistics of the gun. Though the co-volume is an assumed characteristic of propellant gases, it can be evaluated for each propellant composition by closed-vessel testing, as will be shown later. The Noble-Abel equation is a simplified treatment of the complex interaction of gases at high pressure; nevertheless, it is respected for its close agreement with experimental results.

Force Constant

When propellant is burned, it evolves gases at its characteristic flame temperature T_O. If these gases are restrained within the solid walls of a closed-vessel,

* A mole is the value of the molecular weight of a given substance measured in grams. Example: carbon monoxide, formula CO; molecular weight = 12 + 16 = 30, therefore one mole of carbon monoxide weighs 30 grams.

they cannot cool by expansion. Also, as the burning occurs over just a short period of time, the initial heat loss to the vessel walls will be negligible. Thus, at the instant the burning has completed, the gas temperature is still T_0, and the pressure simultaneously reaches a maximum value P_{max}. The equation of state is then

$$P_{max}(V_{cv} - mc) = nR_u T_o$$

where V_{cv} is the volume of the closed-vessel

and m, the mass of propellant.

The value of $\frac{nR_u T_o}{m}$ is a characteristic of each propellant, so the equation can be written

$$P_{max}(V_{cv} - mc) = mF$$

where F is the force constant of the propellant composition.

Measurement of Co-volume and Force Constant

Now, if the closed-vessel is fired with a mass m_1 of a propellant, it will produce a peak pressure P_{max_1}, so

$$P_{max_1}(V_{cv} - m_1 c) = m_1 F$$

A second firing with a different mass, m_2 gives

$$P_{max_2}(V_{cv} - m_2 c) = m_2 F$$

The co-volume and force constant for the propellant can then be found by solving the two equations above.

$$\text{Co-volume, } c = \frac{V_{cv}}{\left(P_{max_2} - P_{max_1}\right)}\left(\frac{P_{max_2}}{m_2} - \frac{P_{max_1}}{m_1}\right)$$

$$\text{Force constant, } F = V_{cv}\,\frac{\left(\frac{1}{m_1} - \frac{1}{m_2}\right)}{\left(\frac{1}{P_{max_1}} - \frac{1}{P_{max_2}}\right)}$$

ENERGY LIBERATED

The gases evolved by a burning smokeless propellant carry with them the energy liberated by the burning process. The amount of energy liberated is given by

$$\text{energy, } E = m\,C_v\,T_o$$

where m is the mass of propellant burnt,

C_v is the specific heat capacity per unit mass of gas at constant volume

and T_o is the absolute flame temperature.

Using the two fundamental gas equations which relate C_v above to C_p, the specific heat capacity per unit mass of gas at constant pressure:

$$C_p - C_v = \frac{nR_u}{m}$$

and

$$\frac{C_p}{C_v} = G,$$

we get

$$C_v = \frac{nR_u}{m(G - 1)}$$

where G is the ratio of specific heat capacities. Putting this into the energy equation gives:

$$E = \frac{nR_u T_o}{G - 1}$$

and as $nR_u T_o$ equals mF,

$$E = \frac{mF}{G - 1}$$

If, at some instant t during the firing sequence of a gun, a fraction z of an initial propellant mass C has burnt, the energy liberated will be

$$E_t = \frac{CzF}{G - 1}$$

ENERGY BALANCE

As with all physical systems, the gun obeys the principle of conservation of energy, so although the energy liberated in the form of propellant gases at the flame temperature is converted into other forms, the total quantity of energy remains the same. Consequently the sum of the converted forms of energy equals the liberated energy throughout the internal ballistic sequence.

$$E_t = E_p + E_g + E_u + E_r + E_b + E_h + E_s + E_f$$

where E_p is the kinetic energy due to the projectile motion,

E_g is the kinetic energy of the propellant gas motion,

E_u is the kinetic energy of the unburnt propellant motion,

E_r is the kinetic energy due to recoil of the gun,

E_b is the heat energy lost to the barrel,

E_h is the residual heat energy in the propellant gases,

E_s is the strain energy used in expanding the barrel,

and E_f is the energy lost in engraving the driving band and overcoming friction in the bore.

Although mathematical models do exist for the energy losses E_g, E_u, E_r, E_b, E_s and E_f, they will for simplicity be summed together here as E_l, so the energy balance equation appears:

$$E_t = E_p + E_h + E_l$$

Now, ignoring the slight rotational energy of the projectile, E_p is simply given by the basic equation for the kinetic energy of a moving mass:

$$E_p = \tfrac{1}{2} M v^2$$

where M is the mass of the projectile

and v is the velocity of the projectile.

However, evaluating the residual heat energy E_h of the propellant gases is more difficult. The propellant gases evolved at the flame temperature cool by expansion as they do work in driving the projectile along the barrel, and are further cooled by contact with the barrel. The average temperature of the gases is some value T, so

$$E_h = Cz\, C_v T.$$

As before $C_v = \dfrac{nR_u}{Cz\,(G - 1)}$, so

$$E_h = \frac{nR_u T}{G - 1}.$$

Substituting the Noble-Abel equation

$$P(V - Czc) = nR_uT \text{ gives}$$

$$E_h = \frac{P}{G - 1} (V - Czc),$$

where P is the average propellant gas pressure and V, the volume occupied by the propellant gases is given by:

$$V = V_a + Ax + V_i$$

in which V_a is the initial air space in the chamber,

A is the cross-sectional area of the bore,

x is the distance moved by the projectile,

and V_i is the increase in volume due to the diminishing size of the propellant granules as they burn.

As $V_i = \frac{Cz}{d}$, where d is the density of the propellant, the residual heat of the gases can now be written

$$E_h = \frac{P}{G - 1} \left(V_a + Ax + Cz \left(\frac{1}{d} - c\right) \right)$$

By substituting the equations for E_t, E_p and E_h, the energy balance equation can now be written

$$\frac{CzF}{G - 1} = \tfrac{1}{2}Mv^2 + \frac{P}{G - 1} \left(V_a + Ax + Cz \left(\frac{1}{d} - c\right) \right) + E_1.$$

For convenience, this is now rearranged to the final form:

$$P = \frac{CzF - (G - 1)\left(\tfrac{1}{2}Mv^2 + E_1\right)}{V_a + Ax + Cz\left(\frac{1}{d} - c\right)} \qquad \text{(Equation 3)}$$

Assuming that E_1 has been substituted by suitable equations, all the parameters in this rearranged energy balance equation are known except for P, z, v and x.

EQUATIONS OF MOTION

Moving on now, we require equations which govern the motion of the projectile. One of Newton's equations of motion states

$$Ma = F_0$$

That is to say, mass multiplied by acceleration equals force. Within the gun, the force acting on the projectile,

$$F_0 = A(P_p - P_a) - F_r$$

where A is the cross-sectional area of the bore,

P_p is the pressure acting on the base of the projectile,

P_a is the atmospheric pressure,

and F_r is the frictional force between the projectile and bore.

Owing to both the inertia of the gases and unburnt propellant, and their friction with the bore surface, the projectile base pressure P_p is somewhat less than the average pressure P. An expression exists which takes this into account and so allows P_p to be calculated once P is known. *

The frictional force F_r is owed to the unrelieved stresses acquired as the driving band engraves into the rifling. Simple mathematical models of the frictional force impose a shot-start pressure which has to be exceeded before the projectile is allowed to move, and apply a constant resistance force for the remainder of the firing sequence. Sophisticated models calculate the energies required to stress and yield the driving band during the engraving process, and use the residual stress once the engraving has completed to find the sliding frictional force.

Writing acceleration as the change of velocity with time t,

$$a = \frac{dv}{dt},$$

then

$$M \frac{dv}{dt} = A(P_p - P_a) - F_r$$

where M is the mass of the projectile.

Integrating this equation with respect to time gives

$$v = \frac{1}{M} \int_0^t \left(A(P_p - P_a) - F_r \right) dt \qquad \text{(Equation 4)}$$

This equation will be used later in finding the velocity of the projectile throughout the firing sequence. Similarly, the equation

$$\frac{dx}{dt} = v,$$

* Ref. Internal Ballistics, HMSO 1951.

which defines velocity as the change of distance with time, can be integrated to give

$$x = \int_0^t v\, dt, \qquad \text{(Equation 5)}$$

and this will be used to find the position of the projectile.

SUMMARY OF THE INTERNAL BALLISTICS EQUATIONS

1. $f = -\frac{B}{D} \int_0^t P^a\, dt$

2. $z = (1 - f)(1 + kf)$

3. $P = \dfrac{CzF - (G - 1)\left(\frac{1}{2} Mv^2 + E_1\right)}{V_a + Ax + Cz\left(\frac{1}{d} - c\right)}$

4. $v = \frac{1}{M} \int_0^t \left(A(P_p - P_a) - F_r \right) dt$

5. $x = \int_0^t v\, dt$

CLASSICAL SOLUTIONS

Until the advent of computers, these equations were solved by mathematical analysis. However, the continuous natures of mathematical functions are poorly able to model the discontinuous processes of internal ballistics, so the five equations were simplified to suit the mathematics. Typical simplifications included taking $a = 1$, imposing a shot-start pressure instead of modelling the engraving process, and taking a constant resistance force to mimic the real frictional resistance force. Further simplifications were made to the equations that constitute E_1, though this was often done because the underlying physical processes were not fully understood. Analytical solutions, such as "A System of Internal Ballistics 1945" by Hunt, Hinds et al of RMCS, were capable of predicting the performance of the contemporary guns with considerable accuracy. However, as the performance of modern guns has increased, the effect of variations in a, F_r and E_1 has become highly significant. It is here that the computer, with its ability to model complex processes, has in recent times ousted the classical analytical models.

COMPUTER MODELLING

A working computer model of the gun has three advantages over the analytical models. It allows realistic data and functions to be included that model discontinuous processes such as ignition and engraving. Its flexibility allows the design of the modelled gun system to be changed at will. And its fast computing speed

enables rapid predictions of gun performance. Clearly, the ability to experiment with gun system designs without having to fire a shot has made computer simulation one of the gun designer's most important tools.

The Lumped Parameter Model

The two main types of computer methods used for internal ballistic modelling are the lumped parameter and the finite difference methods.

Lumped parameter models are in effect numerical solutions to the equations that used to be solved by analytical methods. They are so named because the equations that describe the behaviour of the propellant gases treat the gases as a homogeneous fluid of arbitrary shape. The only term that accounts for any variation in state within the gases at any given moment is the equation for P_p, which estimates the fall in pressure from the gases as a whole to the projectile base. A more accurate model would determine the state of the gases throughout the gun, which in fact is what finite difference models do, though these will be described later.

An essential requirement when modelling such systems as the gun by use of digital computer is that the action described is resolved into discrete steps. The fundamental step that is usually used is a small increment in time, Δt. This in turn requires that integral relationships such as equation 1,

$$f = -\frac{B}{D}\int_0^t P^a \, dt,$$

have to be written as summations, so

$$f = -\frac{B}{D}\sum_0^t P^a \, \Delta t.$$

Similarly, equations 4 and 5 are written

$$v = \frac{1}{M}\sum_0^t \Big(A(P_p - P_a) - F_r\Big) \, \Delta t$$

and

$$x = \sum_0^t v \, \Delta t$$

To ensure that the summations closely approximate the theoretical equations the size of Δt must be kept small. Typically, Δt is taken to be one twentieth of a millisecond.

The flow diagram of a computer program into which the five equations fit is shown in Fig. 2.36. The computations iterate about the program, incrementing by Δt each time, until the projectile either exits from the barrel or stops owing to insufficient propellant.

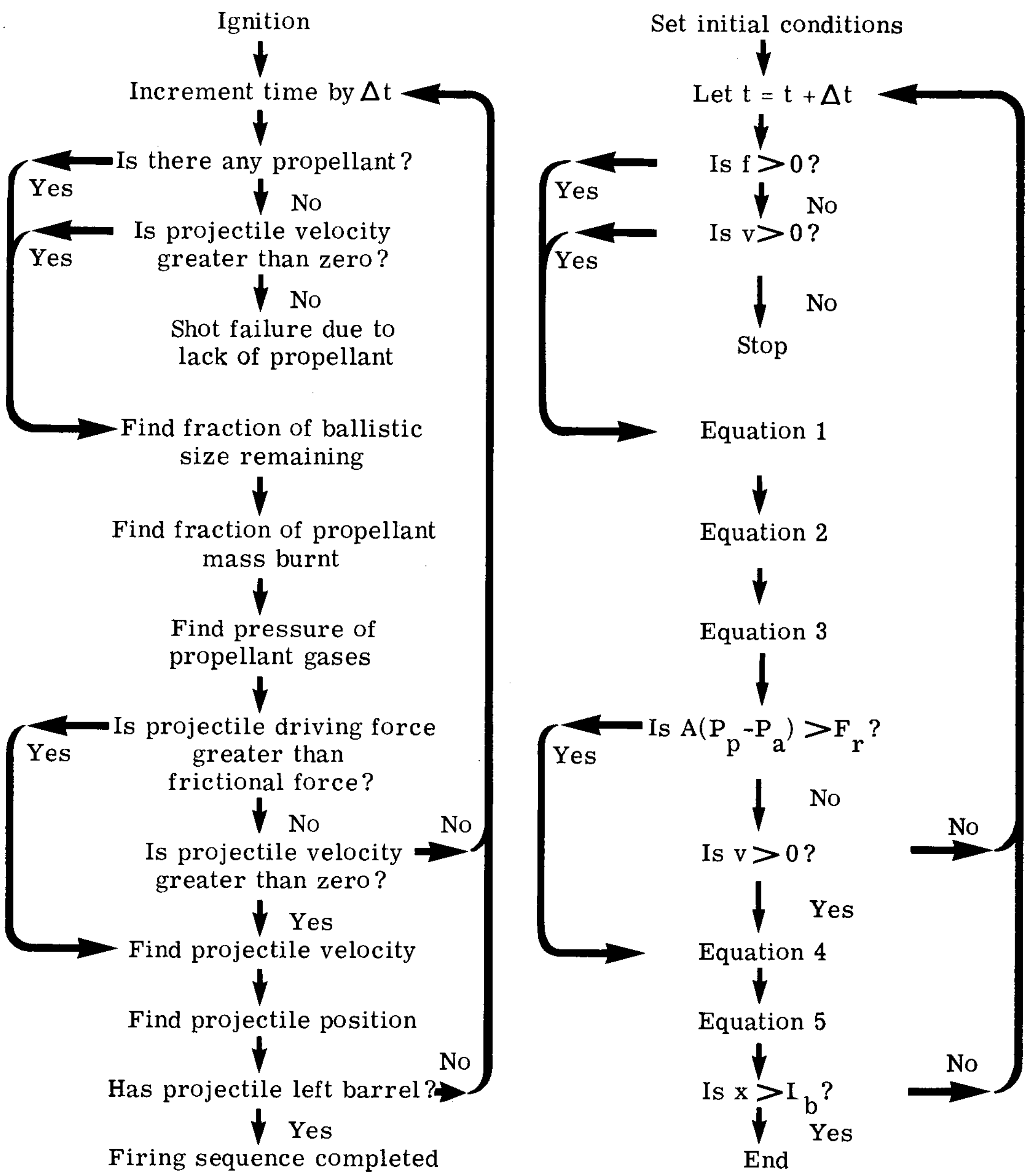


Fig. 2.36 Flow diagram for a lumped parameter computer model of internal ballistics, with description given on left hand side

For accurate modelling it is necessary to make the discrepancy between the summations and theoretical equations negligibly small. One method of achieving this would be to make Δt extremely small, however the program would then take an unacceptably long time to run. The practical alternative is to apply a predictor-corrector technique,* such as the Runge-Kutta procedure, to estimate the discrepancy and so correct each computation.

The Finite Difference Model

The aim of finite difference models is to compute the pressure and flow of the propellant gases throughout the gun. In practice, large pressure differentials are observed along the length of chambers and bores, whereas the pressure differentials across their diameters tend to be slight. This is owed to the geometry of chambers: the length is generally greater than twice the diameter; and the longitudinal asymmetry, with the projectile at one end and the primer often at the other, promotes asymmetry of pressure.

Finite difference models employ a second fundamental step, this being a small distance Δx separating points along the axis of the gun. The gas in the region of each point is individually modelled by a lumped parameter type model which differs from the previous model in that the influence of surrounding regions of gas have to be taken into account. If required, the non-axial regions can be modelled by a two-dimensional array of points lying on a plane that includes the axis. The slight gain in realism afforded by two-dimensional modelling is often outweighed by a large increase in both model complexity and computing requirements.

The finite difference method is able to model the pressure oscillations that occur between the breech and projectile base, and can be adapted to model the process of flame spread during ignition. Both pressure fluctuation and inconsistent ignition are serious problems in modern high performance guns, so this method has considerable advantages over lumped parameter models.

CONCLUSION

This introduction to theoretical internal ballistics has concentrated on the underlying principles of the basic gun and its modelling by computer. Although the vast spectrum of research and modelling is not covered here, there are numerous reference books and papers to which the reader can turn for further study.

There is now a considerable research effort in internal ballistics; it is one of the most significant areas of advancement in ballistics, and its importance is growing with the search for ever increasing gun performance.

* Ref. Numerical Methods for Scientists and Engineers, McGraw-Hill, 1962.

SELF TEST QUESTIONS

QUESTION 1 What is the relationship between burning rate and rate of regression?

Answer ....

....

QUESTION 2 How can the internal ballistic performance of a gun be maintained if the propellant burning rate constant is halved?

Answer ....

....

....

QUESTION 3 If C is the total mass of propellant and z is the fraction of propellant volume burnt, what is the mass of gas liberated?

Answer ....

QUESTION 4 When the frictional force acting on the projectile exceeds the propelling force, what does the projectile do if it is a) stationary, b) moving?

Answer a) ....

b) ....

QUESTION 5 What are the advantages of finite difference methods in computer modelling of internal ballistics?

Answer ....

....

QUESTION 6 The Noble-Abel equation alone cannot completely describe the state of the gas within a gun; why is this so?

Answer ....

....

....

ANSWERS ON PAGE 202

3.

Intermediate Ballistics

DEFINITION OF INTERMEDIATE BALLISTICS

Intermediate ballistics is defined as the study of the transition from internal to external ballistics, which occurs in the vicinity of the gun muzzle. In the case of recoilless guns, the study is extended to the region of the recoil jet nozzles.

The distribution of energy at muzzle exit can be simplified down to:

Motion of projectile	30%	75% Released from gun
Energy of propellant gases	45%	
Heat retained by gun	25%	

Roughly three quarters of the available energy passes through the muzzle; the majority of it is carried by the propellant gases in the form of heat, pressure and motion. After muzzle exit, the behaviour of these gases has considerable influence on the projectile and gun motions; they also give rise to the effects known as blast and flash. At this stage the gas flow takes a distinctive form; it is worth describing the features of it before considering the additional effects that a projectile has as it exits from the muzzle and passes through the gas flow field.

THE GAS FLOW FIELD NEAR A MUZZLE

The release of high pressure gas from a muzzle causes turbulence as it mixes with the ambient air. The resultant pressure waves radiate at the speed of sound as noise. The speed of sound through the muzzle gas flow and surrounding air varies according to the types and state of the gases present. It is roughly given by the equation:

$$\text{Speed of sound in gas, } a_g = \sqrt{GRT}$$

where G = Ratio of specific heats of the gas mixture

R = The gas constant, 287 J/kg K

T = Absolute temperature of the gas.

For air under normal conditions

$$G = 1.404$$

and $$T = 288\text{K}\,(15^{\circ}\text{C}),$$

so the speed of sound in air is 340 m/s.

By substituting the gas equation RT = P/d into the speed of sound equation gives

$$\text{Speed of sound in gas, } a_g = \sqrt{\frac{GP}{d}}$$

where P = gas pressure

and d = gas density

In the vicinity of a gun muzzle during firing, the gas temperature, pressure and density vary considerably, and so the speed of sound also varies. In addition the mixture of gases produced by the burning propellant is very much different from that of air: the resultant differing value of produces further variations in the speed of sound. Although a shock wave is simply an intense sound wave, the momentary rise of the gas temperature as a shock wave passes induces an increase in the speed of sound; consequently shock waves travel faster than sounds of lower intensity. Conversely, a shock wave can be defined as a sound wave of sufficient intensity to self-induce a velocity significantly greater than that predicted by the speed of sound equation.

Noise generated by the turbulent mixing of gases near the muzzle travels both away from the muzzle and towards it. If high pressure gas is suddenly released from the muzzle, which occurs when a projectile exits from a gun, the outgoing noise primarily takes the form of an abrupt increase in pressure known as a blast shock wave. This wave travels away from the gun at speeds slightly greater than the speed of sound and is heard as a sonic bang. The ingoing noise forms a shock wave which travels towards the muzzle against the flow of the gas. Near the muzzle, the speed of the shock wave may equal the speed of the gas flow; when this happens the ingoing shock wave makes no headway and so forms a quasi-static shock wave. This shock wave is bottle shaped, and is sometimes referred to as a 'bottle shock'. The curved sides of the bottle shock that extend from the muzzle are called the barrel shock, and the almost flat base to the bottle shock is called a Mach disc. The size of the bottle shock increases as the outflowing gas velocity increases. As the gas velocity falls the bottle shock shrinks and eventually disappears into the muzzle.

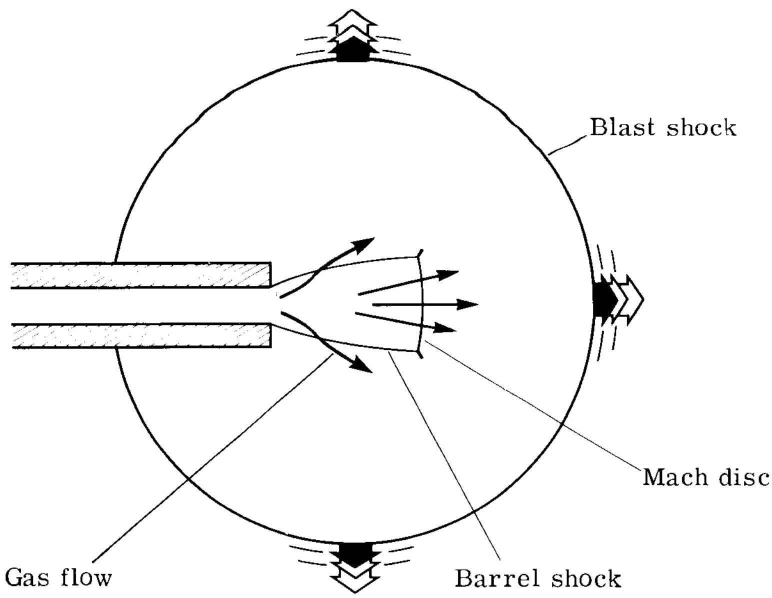


Fig. 3.1 Shock waves formed by the release of high pressure gas from a muzzle

MUZZLE GAS FLOW FIELD DURING FIRING

The muzzle gas flow field during firing consists of two phases: the precursor blast field that preceeds the projectile exit from the muzzle, and the main blast field that follows as the high pressure propellant gases are ejected into the air. In recoilless guns, the blast fields are dominated by the rearward jet of propellant gases from the breech; the flash and blast produced by the jet is similar to that produced by the main blast field of a conventional large calibre gun.

Before Projectile Exit

As the projectile accelerates along the bore, it pushes ahead of it a column of air augmented by any leakage of propellant gases past the projectile. A shock wave forms just ahead of the projectile, travels along the bore, and is released as a near-spherical precursor blast shock at the muzzle. Once the outflowing air velocity is sufficient, a small bottle shock will form about the muzzle, growing in size as the flow velocity increases.

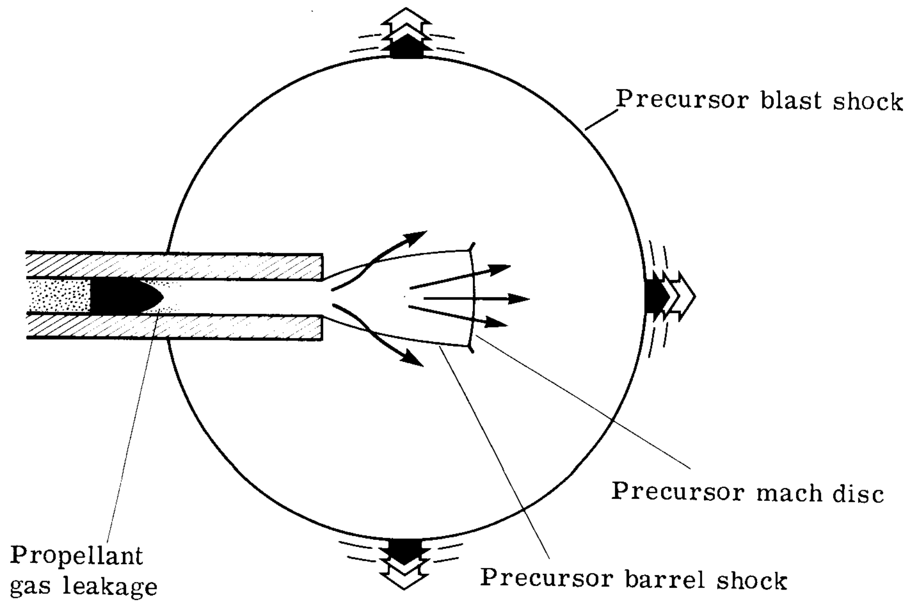


Fig. 3.2 Shock wave formation before projectile exit

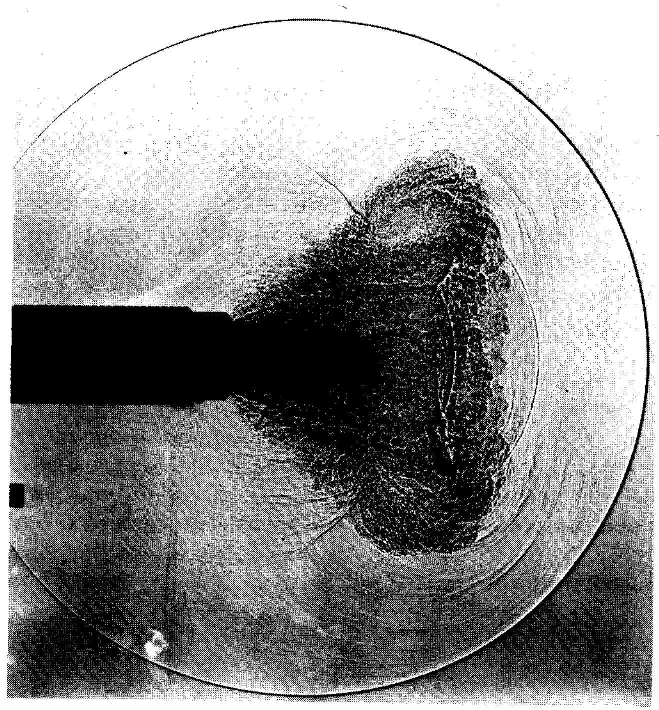


Fig. 3.3 The precursor blast field of a 5.56 mm calibre rifle

After Projectile Exit

The projectile will then emerge, and once the projectile gas seal has passed the muzzle, the high pressure propellant gases will be released into the atmosphere, so generating a powerful blast shock. Initially the blast shock is highly non-spherical as it is distorted by the presence of the projectile and the high velocity flow of the propellant gases. The propellant gases rapidly expand, accelerating to velocities much greater than that of the projectile, so that shock waves form around the base of the projectile, rather as though the projectile is moving backwards. This apparent reverse gas flow provides slight additional acceleration of the projectile for several calibres distance beyond the muzzle. Also, the muzzle gas flow can have an adverse effect on the accuracy of the gun by causing abnormal yawing of the projectile.

A new large barrel shock and Mach disc then forms around the muzzle. As the velocity of the outflowing gases slows, the barrel shock and Mach disc shrinks in size; then the remaining Mach disc enters the muzzle and becomes a rarefaction wave that travels back along the bore. Providing the projectile is fully supersonic it will in the meantime pass through the blast shock. Owing to its high intensity, the blast shock travels faster than the speed of sound and so tends to catch up with the less intense precursor blast shock.

The figures below show three phases in the development of the blast field after projectile exit.

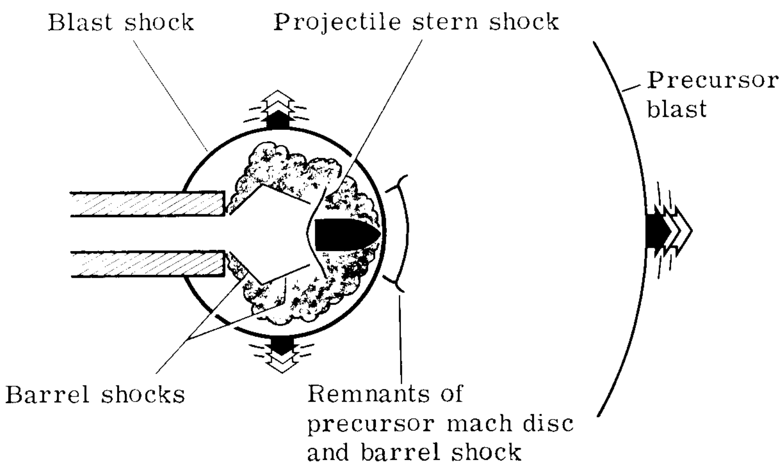


Fig. 3.4 The initial formation of shock waves shortly after projectile exit

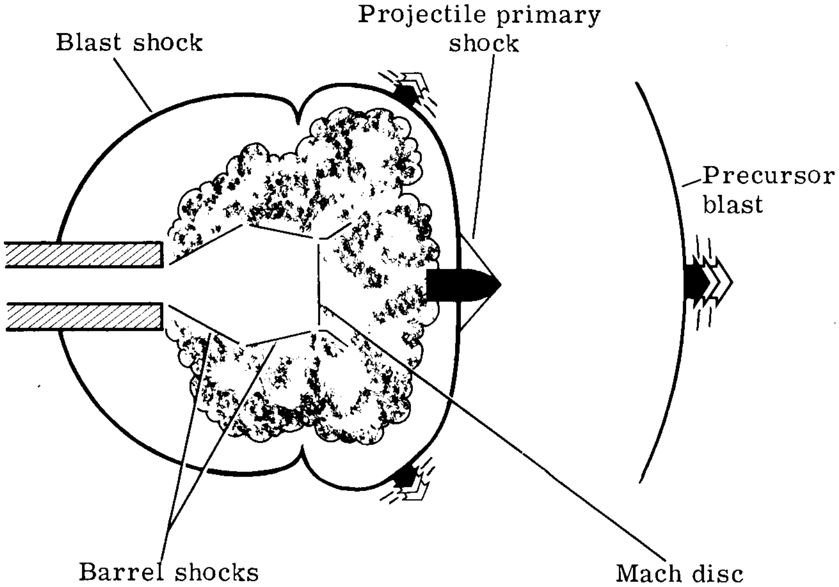


Fig. 3.5 Expansion of the blast field

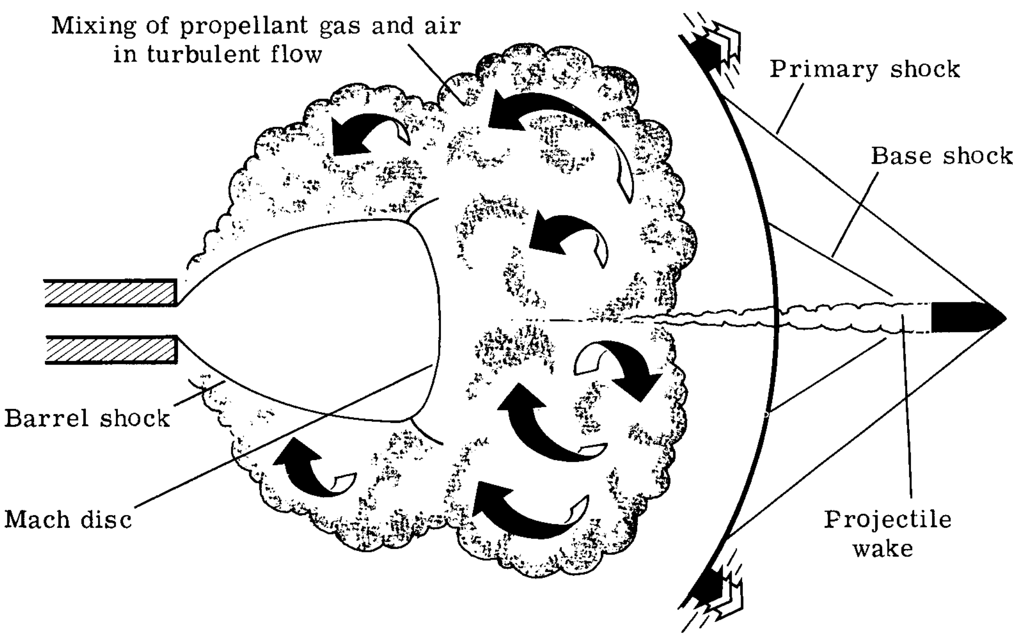


Fig. 3.6 Final phase of the blast field before contraction of the bottle shock and Mach disc

FLASH

Flash is the light emitted in the vicinity of the muzzle by the hot propellant gases and the chemical reactions that follow as the propellant gases mix with the surrounding air.

Before projectile exit, a slight preflash may occur owing to hot gases and particles that have leaked past the projectile. Following muzzle exit, the temperature of the propellant gases is generally sufficiently high to emit visible radiation known as primary flash. As the gases rapidly expand and cool they continue to emit a relatively faint muzzle glow; they are then recompressed as they pass through the Mach disc, and the consequent high temperature produces intermediate flash. Ignition of the hot combustible gases, mainly hydrogen and carbon monoxide, may then follow as they mix with oxygen in the surrounding air; secondary flash, the brightest form of flash, is produced by the ensuing large flame. In small calibre weapons, both the temperature and density of combustible gases in the region of the intermediate flash is usually insufficient to allow ignition, so secondary flash does not occur. Finally, hot particles and remnants of burning propellant may appear as a long streak of light in the wake of the projectile.

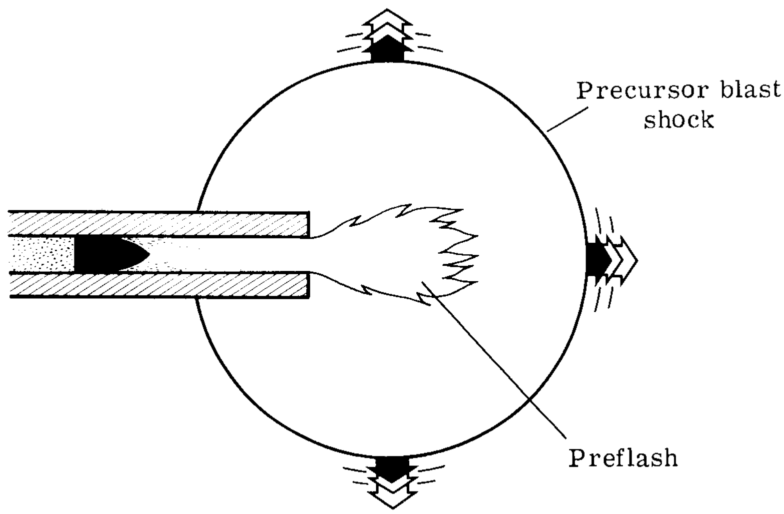


Fig. 3.7 Preflash

For military purposes flash, and especially secondary flash, is undesirable at night as it will indicate the positions of guns to the enemy and will also temporarily blind the gun crews. The three methods of flash suppression are muzzle devices, choice of propellant and propellant additives.

Flash suppression devices are usually designed to suppress the intermediate flash. The reduction of intermediate flash can also suppress the ignition of the secondary flash that occurs in the case of large calibre guns.

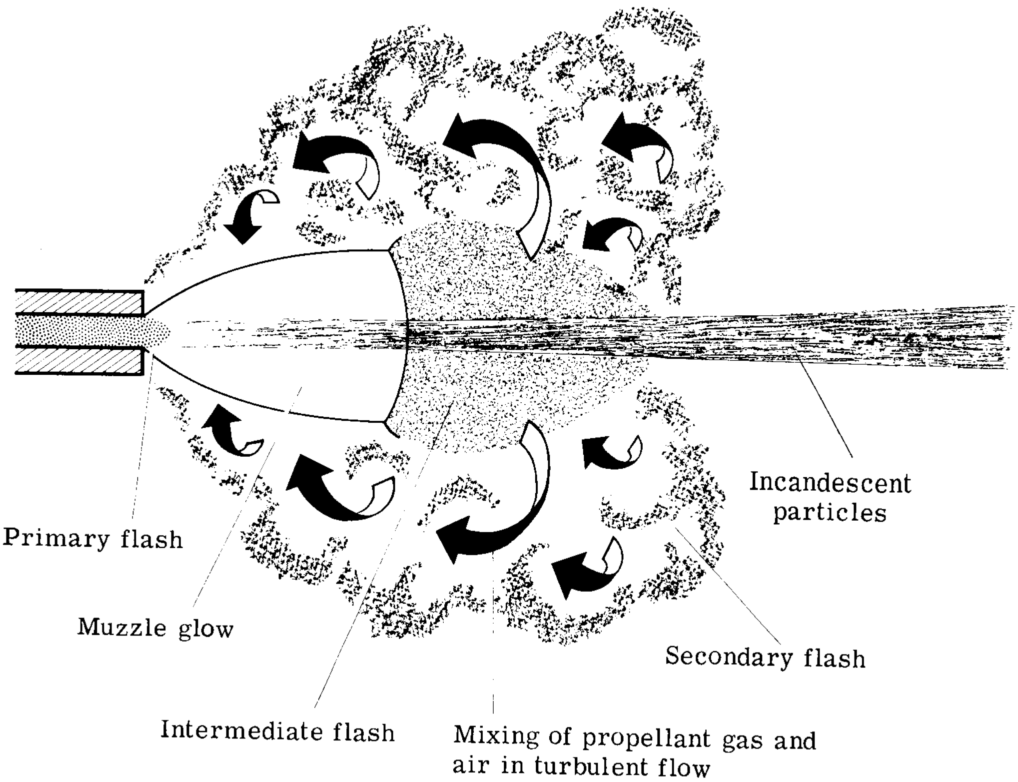


Fig. 3.8 Flash

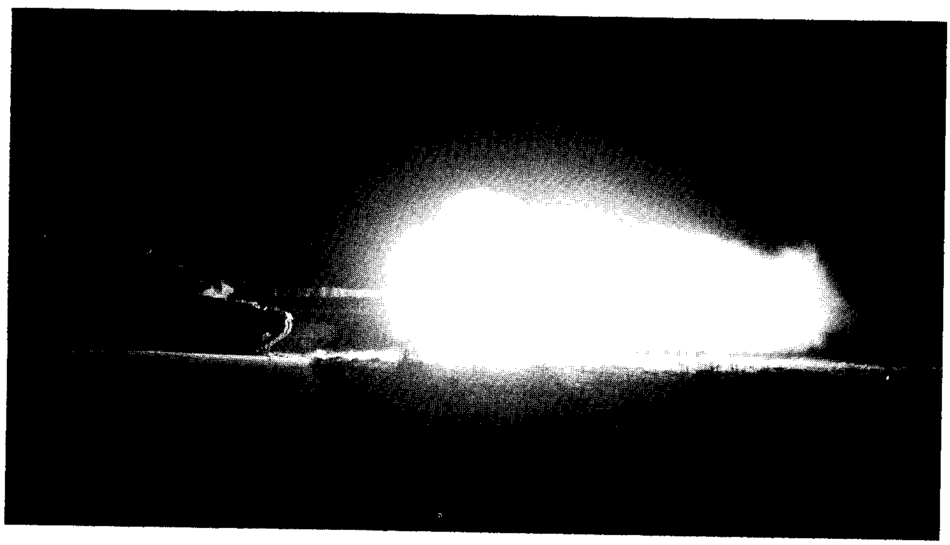


Fig. 3.9 The intense secondary flash of a 120 mm calibre Chieftain tank gun

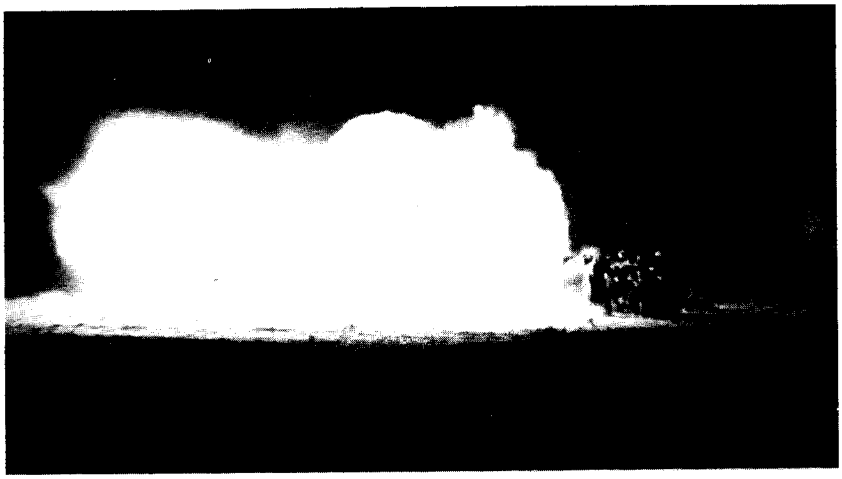


Fig. 3.10 The intense secondary flash from the breech nozzle of a 120 mm calibre Wombat recoilless gun

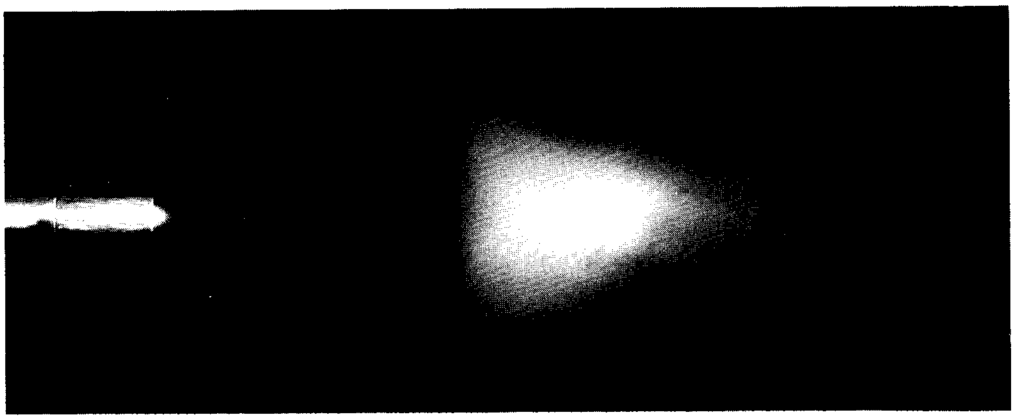


Fig. 3.11 The primary and intermediate flash of a 7.62 mm calibre rifle showing the total absence of secondary flash

The simplest form of suppressor is the flash hider: this is a device which surrounds the primary flash and hides it from all directions except the line of fire. However, as primary flash tends to be insignificant, the original intended purpose of flash hiders is usually ineffective. Modern flash suppressor devices are often referred to as flash hiders, but their real purpose is to disperse or break up the barrel shock and Mach disc so that the intermediate flash is reduced. The three types of flash suppressor devices commonly used are the conical tube, slotted tube, and bar type. The slotted tube and bar type employ a number of slots or prongs protruding forward around the muzzle: an odd number is usually chosen to avoid oscillations of the muzzle gas flow that can interfere with the motion of the projectile. (See Fig. 3.12).

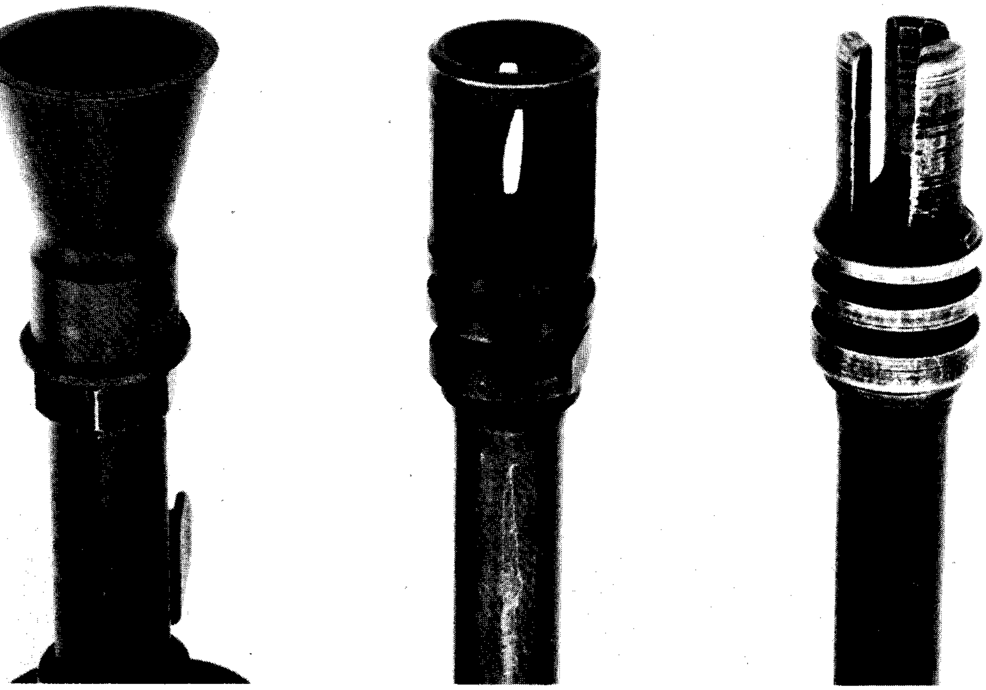


Fig. 3.12 Conical, slotted tube and bar type flash suppressors on 5.56 mm calibre rifles

In large calibre guns, the most effective method of secondary flash reduction is to use a propellant which evolves a large proportion of inert nitrogen gas at relatively low temperatures. Such a technique will dilute the combustible fraction of the propellant gases and reduce the overall temperature. It is achieved by triple-base propellants owing to their large proportion of nitrogen rich nitroguanidine.

Flash can also be reduced by the addition of potassium and sodium salts to the propellant, especially potassium sulphate or nitrate, potassium cryolite and sodium cryolite. Their action is not fully understood, but is known to inhibit the formation and burning of hydrogen gas. However these additives are generally not used because they increase the amount of smoke produced which provides a strong unwelcome firing signature in daytime.

BLAST

The term blast encompasses the effects produced by gas pressure waves in both intermediate and terminal ballistics. The most familiar feature of blast from a gun is the noise produced by the release of high pressure propellant gases into the atmosphere when the gun is fired. Close to the gun, the blast can be sufficiently intense to cause hearing damage and, in extreme cases, injury to lungs

and other soft tissues. The terminal ballistic blast effects produced by explosive projectiles is discussed in Chapter 5.

Problems due to muzzle blast are normally restricted to military weapons and usage. Gun crews are subjected to intense blast noise, often in excess of 140 decibels, which can cause hearing damage. Some form of hearing protection is normally used to avoid temporary hearing loss, or permanent damage in cases of extreme blast or repetitive firings. Excessive blast noise is a particularly serious problem with hand-held anti-tank recoilless guns as the blast from the rear of the tube is severe, and the firer's head has to be held close to the barrel to allow aiming of the weapon. In such cases the blast can not only be heard, but also felt as a blow when it strikes the body. Anticipation of the violent blast the soldier will suffer on firing can markedly reduce his ability to use the gun efficiently if he is not thoroughly trained to it.

Blast noise is further undesirable in military weapons as it indicates gun positions to the enemy. Most guns fire supersonic projectiles so, even if a gun is virtually silent in operation, the shots are still likely to be heard owing to the shock waves generated by the projectile during flight. However, any reduction in muzzle blast will help to avoid identification of the shot's source.

There are two main sources of muzzle blast: blast shock waves and flash blast. The sudden release of high pressure propellant gases that follows the projectile muzzle exit generates an inevitable blast shock wave. Flash blast is produced by the rapid heating and consequent expansion of gases within the secondary flash; it is most prominent in large calibre guns, and can contribute up to half of the total noise on firing. Both of these types of blast cause a rise in the gas and air pressure in the vicinity of the muzzle. In the case of blast shock this overpressure spreads outward as a shock wave, producing an abrupt increase in pressure as it travels. Flash blast resembles an intense noise, accompanied by an overall increase in pressure. Close to the muzzle the overpressure is extremely high, and it rapidly falls to safe levels further away. Gun crews and small arms firers often wear ear defenders to guard against overpressures in excess of 0.2% of atmospheric pressure as they are likely to cause hearing damage.

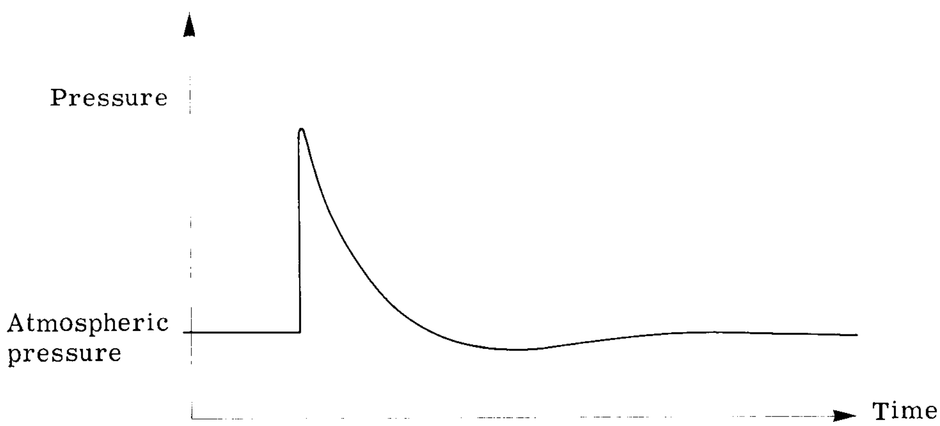


Fig. 3.13 Cross-section of an idealised shock wave

The motion of the projectile and the muzzle gas flow generates a succession of shock waves together with further oscillations of pressure due to turbulence and flash. These variations in pressure can be monitored during firing by fast response pressure gauges, though care must be taken to ensure that their design and orientation do not distort the gas flow, otherwise false measurements will be produced.

The intensity of blast, sound and noise is measured in decibels, a quantity which relates the overpressure produced by a sound to a reference pressure of 2×10^{-5} Pascals, where 1 Pascal equals 1 N/m^2.

Intensity in decibels is given by: $dB = 20 \log_{10} \frac{P_1}{P_0}$

where P_0 is the reference pressure.

For example: find the approximate intensity of an overpressure of 0.2 atmospheres.

As 1 atmosphere roughly equals 1×10^5 Pascals

then .2 " " " 2×10^4 Pascals

$$\text{Intensity} = 20 \log_{10} \frac{2 \times 10^4}{2 \times 10^{-5}}$$

$$= 20 \log_{10} 10^9$$

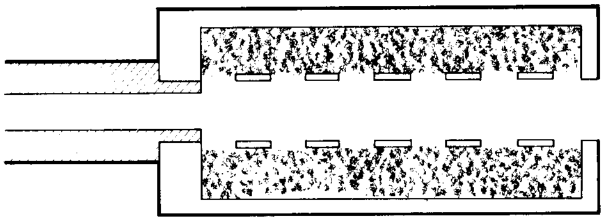
$$\text{Intensity} = 180 \text{ dB.}$$

Intensity in dB	Overpressure in atmospheres
100	0.00002 Approx
120	0.0002
140	0.002
160	0.02
180	0.2
200	2

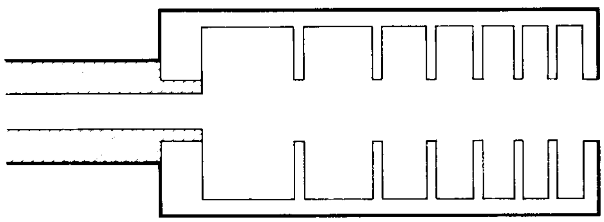
Fig. 3.14 Table of overpressures in atmospheres against intensity of sound in decibels (reference pressure 2×10^{-5} Pascals)

The blast shock from small calibre guns can be suppressed by so-called silencers attached to the muzzle. A silencer never totally silences a weapon, it just reduces the intensity of the muzzle blast; consequently silencers are often known as

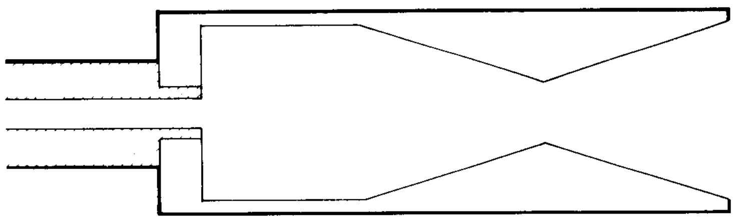
moderators. The three methods of blast suppression commonly employed in silencers are shown below. A silencer may incorporate a combination of these blast suppression methods in its design.



Blast absorption



Blast dissipation



Blast containment and controlled release

Fig. 3.15 Methods of blast suppression

Fig. 3.16 9 mm calibre sten sub machine gun fitted with silencer

In large calibre weapons, the size of silencer required to suppress the blast shock would be impracticably large. Though blast shock is unavoidable, the flash blast from large guns can be suppressed by fitting a flash suppressor, or by adding a flash suppressant to the propellant. Ordinarily, the majority of the blast is directed forwards by the motion of the propellant gases, so the blast that the firers suffer is relatively small. If a muzzle brake has been fitted, some of the propellant gases and the blast they produce are deflected sideways and backwards, so the blast received by the firers is stronger.

There are no clearly defined rules governing the levels above which hearing can be impaired by gun blast. Although recommendations have been made, the suggested limits of safe hearing varies according to the opinions of the recommending council. When assessing the likelihood of hearing damage from gun blast, there are six main factors to be considered:

1. Peak overpressure at the ear.
2. Effective duration of the intense blast.
3. Frequency range of the blast noise.
4. Ear protection used.
5. Repetition of exposure to blast.
6. Personal susceptibility to blast.

The effective duration of the blast can be defined in a number of ways. For example, the blast duration can be defined as the time taken from the initial rise of pressure to the moment that the overpressure falls to a level permanently below one tenth of the peak pressure. The sensitive frequency range of the ear is roughly from 1000 Hz to 6000 Hz; intense noise in this range tends to cause damage and the consequent impairment of hearing will reduce sensitivity in this range first. Mild loss of hearing in the sensitive range often goes unnoticed as the reduction in clarity of most sounds is disguised by the continued ability to hear sounds at other frequencies. Proper use of ear defenders can typically reduce the effective intensity of the blast noise at the ear drum by up to 35 dB. This alone will protect hearing in most cases: the main exceptions are recoilless and tank guns in which the combined effect of intense blast and unavoidably close proximity of the crews can sometimes lead to blast levels in excess of 185 dB; in such cases extra hearing protection should be used. Unless properly fitted, ear defenders and ear plugs are generally far less effective, giving protection of 15 to 25 dB. The ear is usually capable of withstanding short single blasts in excess of 150 dB, but repetitive exposure to even low levels of blast can lead to permanent hearing damage. The restriction on the number of repeat firings and the period of firing depends on several factors, complicated by the circumstances in which guns are used. For example, firing an anti-tank recoilless rocket launcher without ear protection more than once every day would soon cause hearing damage, but a soldier in battle could fire the same gun many times in one day yet not suffer permanent hearing damage as he may never need to fire such a gun again. The ability to tolerate blast noise varies considerably from person to person and so care

should be taken to ensure that adequate protection is used in every case to safeguard the hearing of those most susceptible to damage.

RECOIL

Recoil is the rearward motion of the gun in reaction to the forward motion imparted to the projectile and propellant gases. The forward momentum gained by the projectile and gases is accompanied by an identical gain in rearward momentum of the gun. The speed of the recoiling gun is small in comparison to the muzzle velocity, but the gun is relatively massive and so requires a considerable braking effect to halt its motion. The recoil of small arms is usually acceptable, but for larger weapons recoil can be a serious problem.

Typically, a gun will have attained about half of its final recoil energy when the projectile leaves the muzzle; the remaining half of the recoil energy is gained by reaction to the rapid outflow of gases at the muzzle. Alternatively, a large proportion of the outflowing gases can be deflected backwards by a muzzle brake, and so generate a forward thrust that partially counteracts the recoiling motion of the gun.

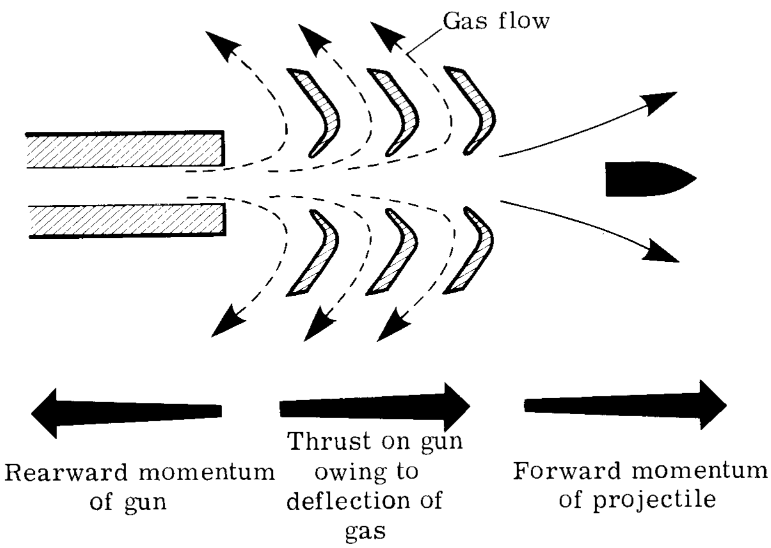


Fig. 3.17 The principle of the muzzle brake

An efficient muzzle brake can reduce recoil by over 50%, though such muzzle brakes are complex, costly and, as has been indicated, can damage hearing. Practical muzzle brakes reduce recoil by about 25% by deflecting some of the gas flow sideways rather than backwards.

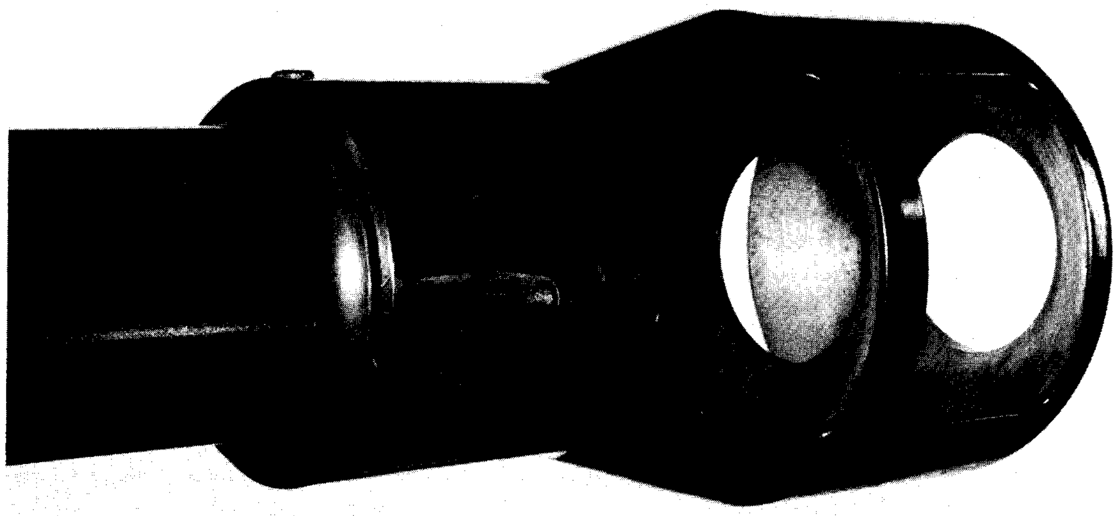


Fig. 3.18 Muzzle brake fitted to British 105 mm calibre light gun

Muzzle brakes can be designed to deflect gas mainly upwards, for example, to control the upward muzzle jump of submachine guns. The prime disadvantage of the muzzle brake is that it increases the blast noise suffered by the firer. It can also subject the fore-end of the barrel to excessive stress. Generally sabotted ammunition cannot be used in guns fitted with muzzle brakes as the complex gas flow at the muzzle interferes with the sabot discarding process, thereby reducing the accuracy of fire. Besides, the sabot is very likely to collide with the muzzle brake.

THE DILEMMA OF INTERMEDIATE BALLISTICS

There are so many variables which complicate the objective study of intermediate ballistics that it is by no means an exact science. Indeed it is an area which can cause gun and ammunition designers considerable problems. This chapter has covered the areas which theory and empirical methods have isolated as affecting design. However there remains no precise methods to predict intermediate ballistic performance, so practical trials are still necessary to assess every new design.

SELF TEST QUESTIONS

QUESTION 1 Name the shock waves produced by the firing of a gun.

Answer ....

....

....

....

QUESTION 2 What effects does the muzzle gas flow have upon the projectile?

Answer ....

QUESTION 3 What is the minimum speed of a shock wave?

Answer ....

QUESTION 4 Which types of flash would you expect from a rifle?

Answer ....

....

QUESTION 5 Which type of blast is not produced by small calibre guns?

Answer ....

QUESTION 6 If an overpressure of 0.08 atmospheres is recorded near a gun, what is the intensity of the blast at that point relative to a reference pressure of 2 x 10^{-5} Pascals?

Answer ....

QUESTION 7 Why is the recoil momentum of a gun greater than the forward momentum imparted to the projectile?

Answer ....

....

....

ANSWERS ON PAGE 202

4.

External Ballistics — Part I

BASICS

Once the projectile has left the gun and the influence of emerging gases, the part of the flight known as external ballistics begins. There are a number of factors which affect the motion of the shell, some associated with the shell itself and others associated with the air through which the shell is moving. The properties of the shell which enter into the problem are mass, calibre, nose shape and spin rate, whilst those associated with the atmosphere are air density, temperature, pressure and viscosity. One of the prime considerations in shell design is accuracy of fire and so some means of stabilisation must be employed. The two most common means of achieving this are spin and fin stabilisation. If the shell is spun the combined action of gyroscopic and aerodynamic effects will cause it to drift sideways. Another significant effect for longer ranges is that due to the rotation of the earth. We consider these factors in more detail later. We begin our study of external ballistics by considering the motion of a projectile under the influence of gravity only, that is, we assume that the projectile is fired in a vacuum.

MOTION IN VACUO

Consider a projectile fired at an angle of elevation θ with a muzzle velocity V metres per second (V m/s). Newton's first law of motion states that a body in motion continues to move in a straight line at a constant velocity V unless acted upon by an external force. Therefore, in the absence of gravity or other forces the projectile in Fig. 4.1 would continue in its initial direction and maintain its muzzle velocity. But, of course, there is an external force, the force due to the Earth's gravitational field, which has the effect of pulling the projectile back towards the centre of the earth with an acceleration of g m/s^2. The value of g varies with distance from the earth, but for short range weapons, such as small arms and field guns, it can be assumed that the gravitational field is uniform, and take a constant value 9.81 m/s^2.

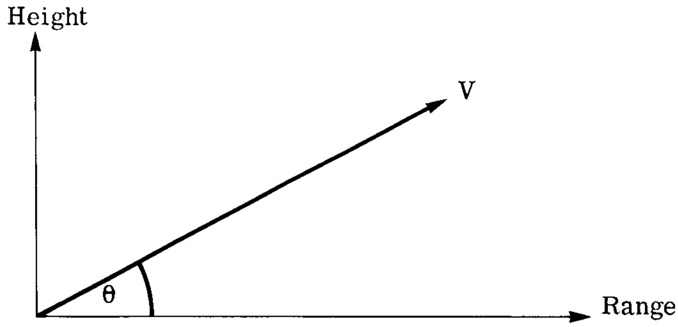


Fig. 4.1 Motion of a projectile under no external influences

We now have the situation shown in Fig. 4.2.

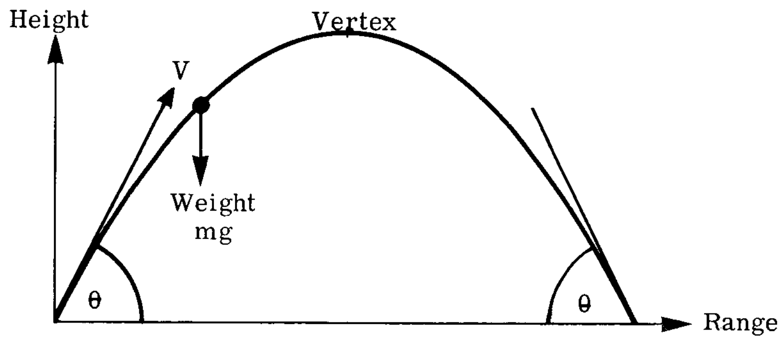


Fig. 4.2 Motion of a projectile under the action of gravity

The characteristics of "in vacuo" trajectories are:

1. The trajectory* is symmetrical about the vertical line through the apex of the trajectory and moves along a curve called a parabola.

2. The whole trajectory lies in the vertical plane containing the line of departure.

3. The range depends upon the muzzle velocity and increases as the angle of departure θ increases up to 45°. For a given velocity, the maximum range is achieved at 45°.

*The trajectory is the path taken by the centre of gravity of the projectile.

4. As θ increases above 45^{o}, the range decreases until it becomes zero at $\theta = 90^{o}$. At this stage, the maximum height is reached and the time of flight is maximum.

5. The angle of arrival equals the angle of departure and the final velocity at the point of impact equals the muzzle velocity. The velocity is a minimum at the vertex.

6. The trajectory is entirely independent of the nature of the projectile, since air resistance is zero.

Although the in-vacuo theory is relatively simple, the mathematics involved in the study of the real trajectory is much more complicated. However, it is worth noting that it is accurate for ballistic trajectories that are predominantly outside the Earth's effective atmosphere, such as those of Inter-Continental Ballistic Missiles (ICBM's), and is reasonably accurate for mortars if the muzzle velocity is less than 250 m/s.

AIR RESISTANCE

Forebody Drag

As the projectile moves through the air, the air is displaced. The energy needed to displace this air causes a continual drain on the initial kinetic energy of the projectile: this loss of energy is called the drag and causes a continuous loss of projectile velocity. The compression of the air, which occurs immediately in front of the projectile, is transmitted to the surrounding air as a pressure wave. In its turn this causes a disturbance which travels through the air at the same speed as sound waves, since sound waves are themselves merely disturbances of the air. When the projectile is travelling at a speed below that of sound, that is below approximately 340 m/s, the disturbances move faster than the projectile and so spread out away from it, as shown in Fig. 4.3.

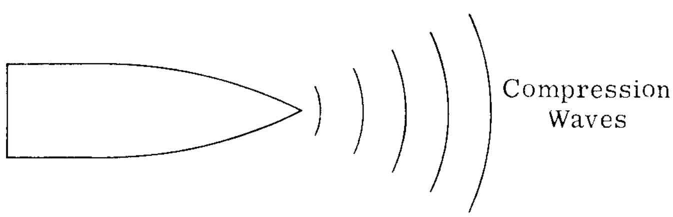


Fig. 4.3 Compression waves at subsonic speed

The generation of these compressed waves causes the projectile to experience "forebody drag". Forebody drag, although considerable, is of greatest significance in the supersonic region. When the projectile is travelling faster than sound no part of the disturbance can escape directly in front of the projectile since it is moving faster than the disturbance. The result is that the compression waves coalesce or "bunch up", and a shock wave is created at the nose of the projectile, as shown in Fig. 4.4.

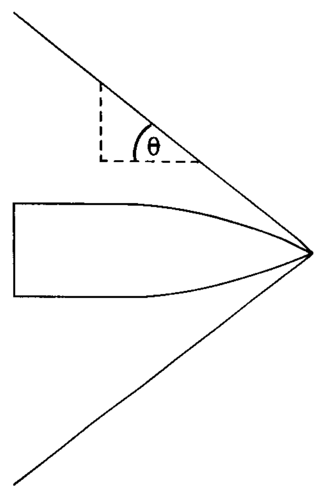


Fig. 4.4 Shock wave at supersonic speed

Shock Waves

It is worth discussing the feature of shock waves in some detail. In general, a conical shock wave is generated which has an angle θ where $\sin \theta = 1/M$. M is the Mach number and is defined as the velocity of the projectile divided by the local speed of sound in air, a, that is $M = V/a$.

Some typical values are:

M	1	1.5	2	3	10
θ (deg)	90	42	30	19	<6

which show how rapidly the shock cone decreases with increase of Mach number. At hypersonic speeds, which is defined as over Mach 5 ($M > 5$), the shock wave

almost follows the shape of the body. With sharp-edged or pointed bodies the shock wave usually originates from the nose of the body, so the shock wave is attached.

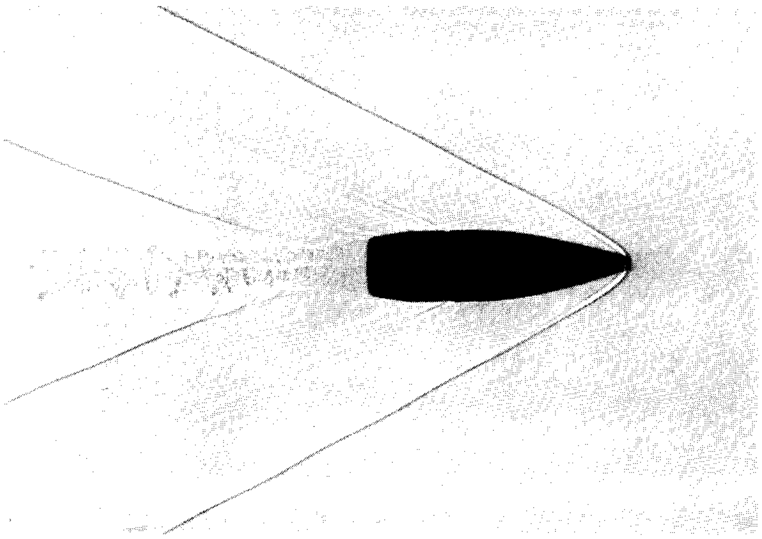


Fig. 4.5 Shadowgraph of 7.62 mm bullet

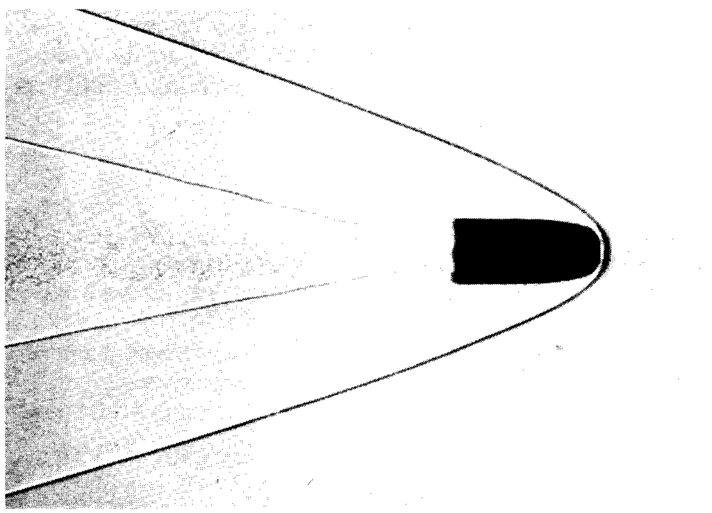


Fig. 4.6 Shadowgraph of high drag training round

If the body has a blunt nose or a larger angle, then the shock wave forms just ahead of the body, so the shock wave is detached.

Fig. 4.7 Shadowgraph of space re-entry model showing detached shock wave

The more blunt the body the higher the shock wave drag and therefore the greater the retardation. This is clearly shown in the flight of space vehicles where the exit profile shape has an ogival or conical cover, which is jettisoned before re-entry to reveal a blunt forebody which can dissipate the consequent high thermal energy and give the necessary retardation.

Because we are dealing with finite length bodies travelling supersonically, the disturbance generated must also be finite, and so a bow or nose shock wave is accompanied by a stem or tail shock. This pair of shock waves constitutes the "sonic boom" heard from bullets and shells as well as from supersonic aeroplanes such as 'Concorde'.

Base Drag

It is clear from the shadowgraphs shown above that there is considerable turbulence behind the bodies. This turbulence called the wake, causes a further resistance known as "base-drag". The cause is a region of low pressure immediately behind the projectile which results because the air flow cannot return quickly enough to fill the space behind the projectile. The consequence is a vacuum or suction effect which shows itself in the form of a resistance to motion. Figure 4.8 illustrates the effect of base-drag.

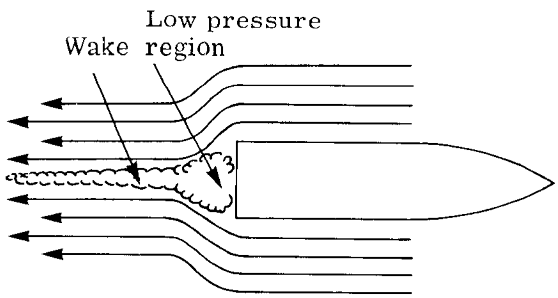


Fig. 4.8 Base drag

Skin Friction

Additional resistance to motion is caused by air adhering to the surface of the projectile; it is known as "skin friction". The mechanism is basically that the air at the surface of the shell is moving at the same speed as the shell; the next layer of air is moving a little more slowly, and so on outwards. Frictional drag, which results, is generally relatively small for most shells; but for larger missiles, the effect becomes more important and certainly has to be considered in rocket calculations. A smooth and polished surface will help to reduce skin friction. The volume in which these viscous forces act is called "the boundary layer".

Excrescence Drag

The final form of resistance called "excrescence drag", arises from protuberances from the shell and can be minimised by eliminating all unnecessary projections on the projectile. Normally only the driving band is unavoidable and this must be carefully designed.

Summarising, the components of air resistance affecting projectiles are:

1. Forebody drag
2. Base drag
3. Skin friction
4. Excrescence drag

The following diagram is for a conventional projectile and shows the relationship between the three main causes of air resistance and the projectile velocity.

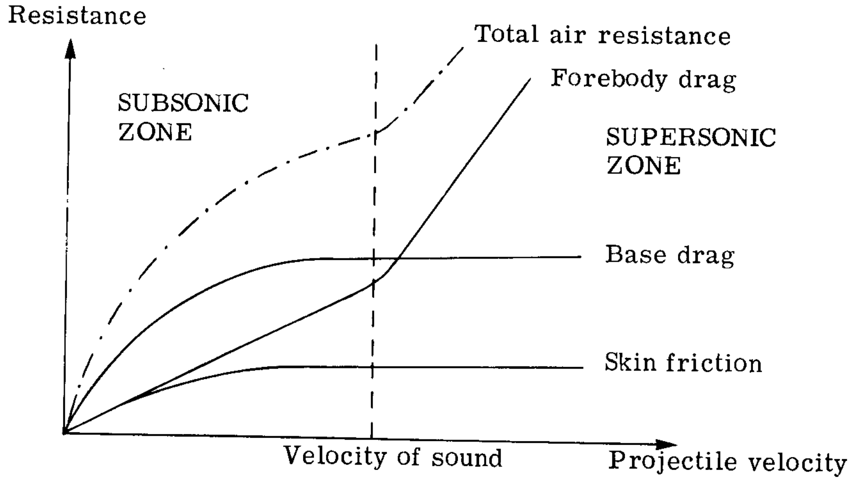


Fig. 4.9 Air resistance

Variations in Drag Values

Forebody drag increases steadily as velocity increases, and a steep rise is noticed as the velocity of sound is approached. The increase is maintained, for a time, at this higher rate in the supersonic zone, but gradually reduces. Base drag increases with velocity until the velocity of sound is reached, but then remains fairly constant. This is because, as the velocity of the projectile approaches the speed of sound, the air pressure behind the base tends to zero, but since the pressure cannot become absolutely zero, any increase in velocity has little effect. Skin friction is normally of least consequence but as mentioned earlier, it can be relatively greater for long thin projectiles.

At this stage it is worth commenting on the transonic zone. Around the velocity of sound there is a zone in which the projectile's behaviour is unpredictable because the air resistance is changing extremely rapidly. This is noticed in the previous diagram, Fig. 4.9, especially as regards forebody drag. In this transonic zone small changes in the projectile's velocity, due perhaps to wind fluctuations, cause marked changes in resistance. It is important that the projectile is steady when it enters the transonic zone. Since it is common for projectiles to be unsteady just outside the muzzle of the gun, it is undesirable to use a charge which gives a muzzle velocity only slightly above the velocity of sound. For this reason, muzzle velocities in the region of the velocity of sound are, if possible, avoided by the designer.

The Form of the Real Trajectory in Air

The main effect of air resistance is to impede the progress of the projectile, both in the rising and falling parts of the trajectory, and to distort the in-vacuo trajectory shown in Fig. 4.2. Air resistance can be considered as a retarding force always acting in a direction opposite to the motion of the projectile's velocity. For the in-vacuo trajectory, the vertical component of the projectile's velocity is retarded by gravity, while the horizontal component is constant.

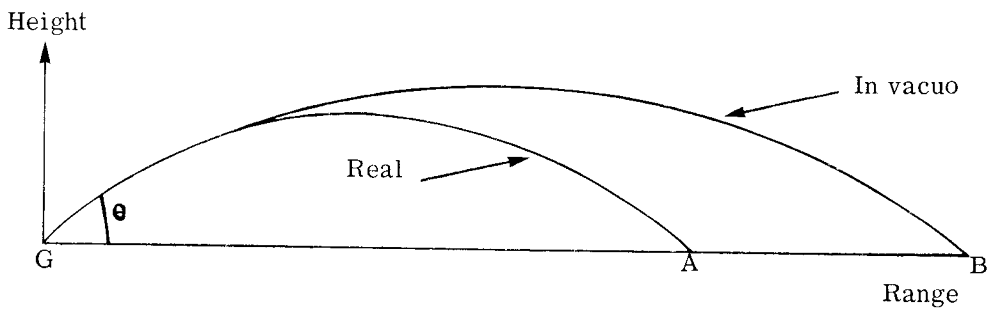


Fig. 4.10 Real and in-vacuo trajectories

Figure 4.10 shows the in-vacuo trajectory and the real trajectory in air for the same projectile and projectile conditions. In air, during the rising portion of the trajectory, air resistance acts in addition to gravity, and the vertical component of the velocity is reduced to zero more rapidly. The horizontal component of the initial velocity is also reduced by air resistance.

During the fall the air resistance acts to oppose gravity so that the time for the fall is greater than the rise. All these effects can be summarised as:

1. The trajectory is no longer symmetrical: the vertex is nearer to the point of impact than to the gun.

2. The vertex height is less than for the corresponding "in-vacuo" trajectory.

3. The angle of descent is greater than the angle of departure.

4. The terminal velocity is less than the muzzle velocity.

5. The smaller the drag coefficient of the projectile the nearer point A is to B.

Before studying the drag coefficient we consider the results in the following table, which show that the presence of the atmosphere modifies considerably the maximum range predicted by the in-vacuo theory.

Type of round	Muzzle Velocity m/s	Maximum Range	
		In-vacuo	In Air
300 mm Mortar	396	16 km	11 km
155 mm FH70	700	49 km	24 km
7.62 mm SLR	840	70 km	4 km

Fig. 4.11 The effect of air resistance on certain projectiles

The Drag Coefficient

The drag coefficient is a convenient way of expressing the ballistic performance of a particular projectile. We have seen that aerodynamic drag can be broken down into a number of components; in general the total drag force can be expressed as:

$$\text{Drag force} = \tfrac{1}{2}\rho\, V^2 A C_D$$

Where ρ is the density of air, V is the velocity of the projectile, A is the cross-sectional area based on the calibre of the projectile, and C_D is the drag coefficient. C_D has a value between 0 and 2 which is dependent upon the streamlining of the projectile and its attitude in flight. Typically a shell shape gives a value of $C_D = 0.3$ while for a blunt body like a training round $C_D = 0.8$. In general the lower the value of C_D the lower the drop in velocity with range. It must be remembered however that C_D is only constant at speeds well below 300 m/s or well above 1500 m/s. When C_D is constant, it means of course that the drag is proportional to V^2. The variation of C_D between 300 and 1500 m/s is discussed later. In recent years it was commonplace to speak of a "ballistic coefficient" when describing the ability of a projectile to fly a particular trajectory or to be compared with the trajectory of another projectile. The term standard ballistic coefficient was defined as $C_O = W/\kappa\sigma d^2$ where W was the projectile weight in lbs, $\kappa\sigma$ was an adjusting term for "shape and steadiness" with a value close to unity, and d the calibre of the round in inches. In practice C_O took a value of between 2 and 3 for a reasonably efficient shell, and it could be seen that if the ratio of W/d^2 was increased then the shell went further for the same muzzle velocity. The value of C_O was used in conjunction with the special tables called P (v) tables to predict the retardation of a particular shell shape at different velocities. However, we now use C_D, in line with modern aerodynamic terminology.

PROJECTILE SHAPE

The air resistance suffered by a shell in flight depends on the shape of the shell; of particular importance are the shapes of the nose and the base.

Calibre Radius Head

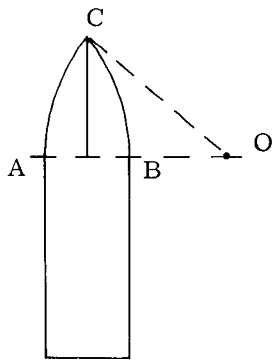


Fig. 4.12 Calibre radius head

Prior to World War I, the shape of the nose was usually a simple ogive. Thus in Fig. 4.12 AC is an arc of a circle drawn about 0 as centre. If OA = n x AB, the shell is said to be of n c.r.h. (calibre radius head). Normally n would be about 2. Modern shells are normally given a more pointed nose of somewhat different shape. The point C is located as before but the actual profile between A and C is an arc of longer radius drawn about a point.

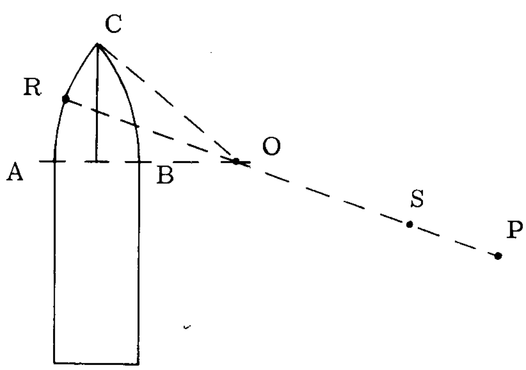


Fig. 4.13 Fractional calibre radius head

AC is bisected at R, and the line RO is produced in the direction of P. A centre of curvature S can be selected anywhere on OP. With S as centre an arc AC of radius SA is struck, and it is this which defines the semi-profile of the head. A head of this kind is defined as a fraction. Thus a 5/10(crh) is a head the length of which corresponds to that of a simple ogive of radius 5 calibre. The curvature is 10 calibre ie SA is ten projectile diameters.

METHODS OF REDUCING RESISTANCE

Boattailing

Aerodynamic drag has a significant effect on the range and performance of high speed projectiles, so it is desirable to design a projectile with the minimum of drag. It may cause other problems such as instability but, as in most design problems, a "trade off" is required. Below the speed of sound base drag and skin friction are the dominant causes of resistance. To reduce base drag, boattailing is often used. A boattail is usually a truncated cone of the order $\frac{1}{2}$-$\frac{3}{4}$ calibre in length; it reduces the pressure drop in the wake of the projectile by allowing the air to flow in and around at the base.

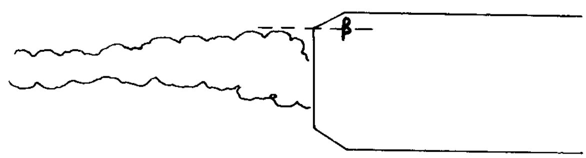


Fig. 4.14 Boattail design

The optimum value of the angle β has been found to be about 7.5°. Recently new shapes[1] have been proved to be superior to conical both from the base drag aspect as well as aerodynamic stability at transonic speeds. These shapes are formed by slicing planes at an angle to the cylindrical body in the base regions. They result in square and triangular base shapes. Tests have shown that a longer range is acquired with boattailing than a flat base projectile of the same weight and velocity. Boattailing is undoubtedly effective at subsonic speeds and in fact it was originally designed to improve the long range subsonic performance of machine guns. However, there is a difference of opinion concerning its effectiveness at supersonic speeds;[2] reports that an increased wake dispersion results when projectiles are supersonic, with a subsequent decrease of performance. We know that there are disadvantages of boattailing due to manufacturing problems, greater barrel wear and erosion. On the other hand[3] provides us with results that show that there is a substantial increase in range because of the boattail effect. It is found on many supersonic projectiles, but a complete justification of its use has not yet been formed.

Nose Shape

Subsonically, forebody drag can be significantly reduced by any suitable smooth nose shape. The graph at Fig. 4.15 shows the relationship between the drag coefficient C_D with an increase in speed mainly in the subsonic region, for three various shapes of nose.

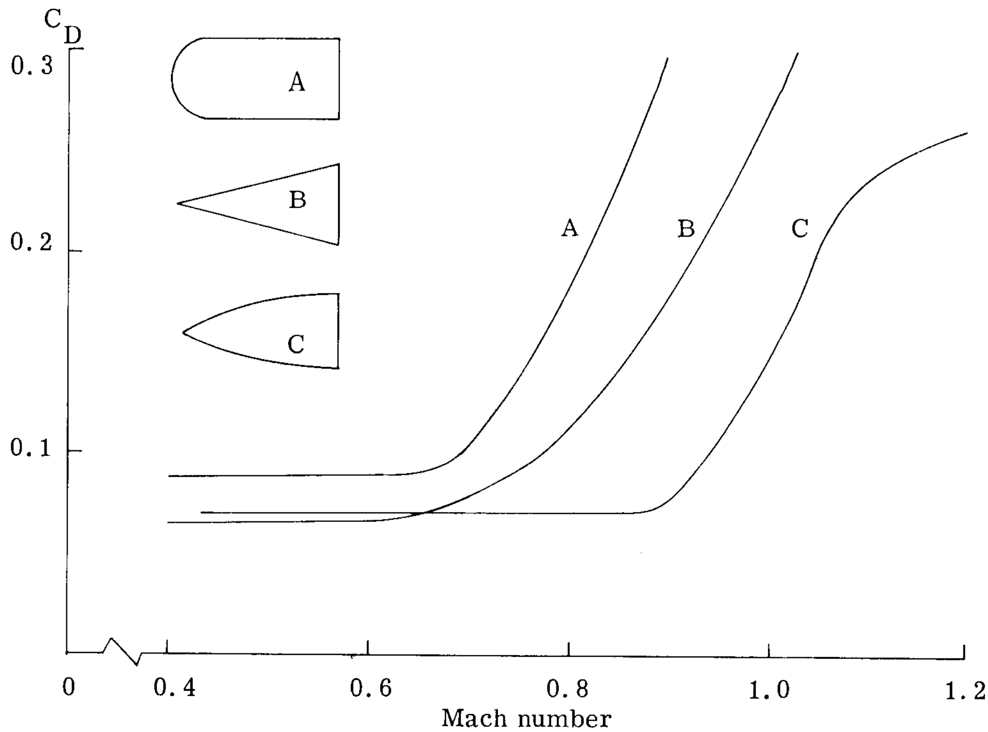


Fig. 4.15 The variation of C_D with speed in the subsonic region for different nose shape projectiles

At supersonic speeds, the effect of base drag and skin friction components are similar to those experienced below the sound barrier; but the shape of the projectile has a great influence on the forebody drag. Mathematical methods are available[4] to calculate the shape of least resistance of a projectile if the dimensions are known.

In general, the longer the head the smaller the forebody drag at supersonic velocities. The total length of a spin stabilised projectile lies between 4.5 and 6 calibres of which between 2.7 and 4 calibres are available for the head. It must be pointed out however that for important technical reasons, the total length has generally to be restricted to 5 calibres. Only if tactical reasons absolutely required an exceptionally thin projectile would this value be exceeded. Figure 4.16 illustrates the variation of C_D the drag coefficient with speed for high velocities, for two different objects, while Fig. 4.17 shows the variation of C_D with speed for a typical shell.

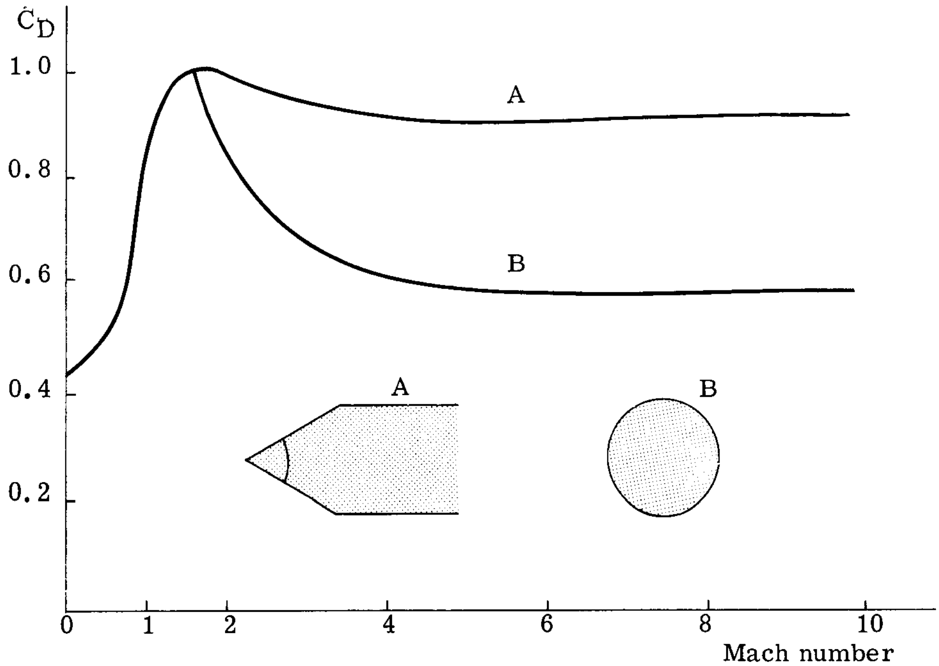


Fig. 4.16 The variation of C_D with speed for two differently shaped projectiles

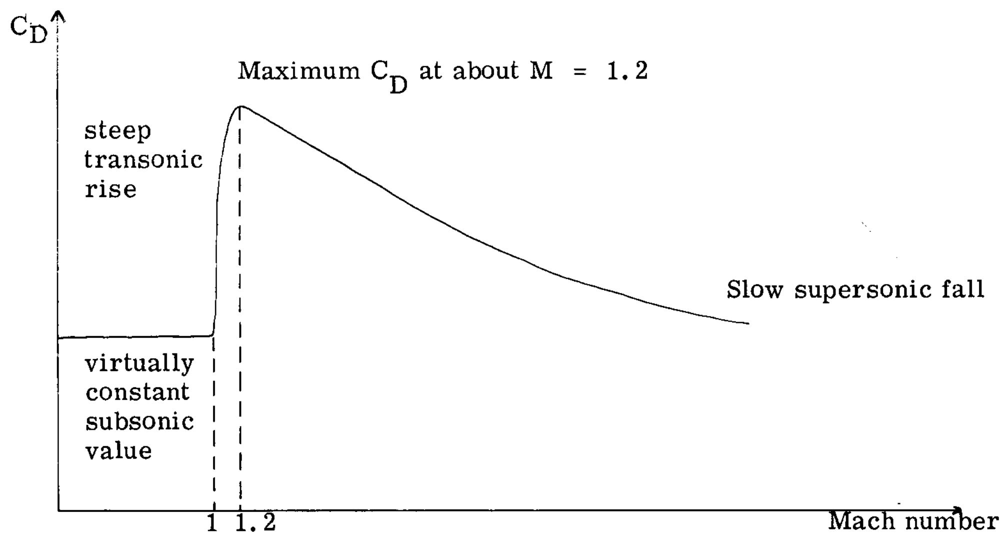


Fig. 4.17 The variation of C_D with speed for a typical shell

Base Bleed

A practical method of reducing drag, especially for larger projectiles is "base bleed". This process involves burning a small quantity of propellant which is fitted into the rear of the projectile. The propellant burns at low pressure and generates a jet of gas at the base, which in turn increases the pressure behind the projectile base and can reduce base drag by up to 50%. Range increases of between 10% and 30% have been achieved with this method. The graph below illustrates the marked effect of base bleed on a 120 mm experimental projectile.

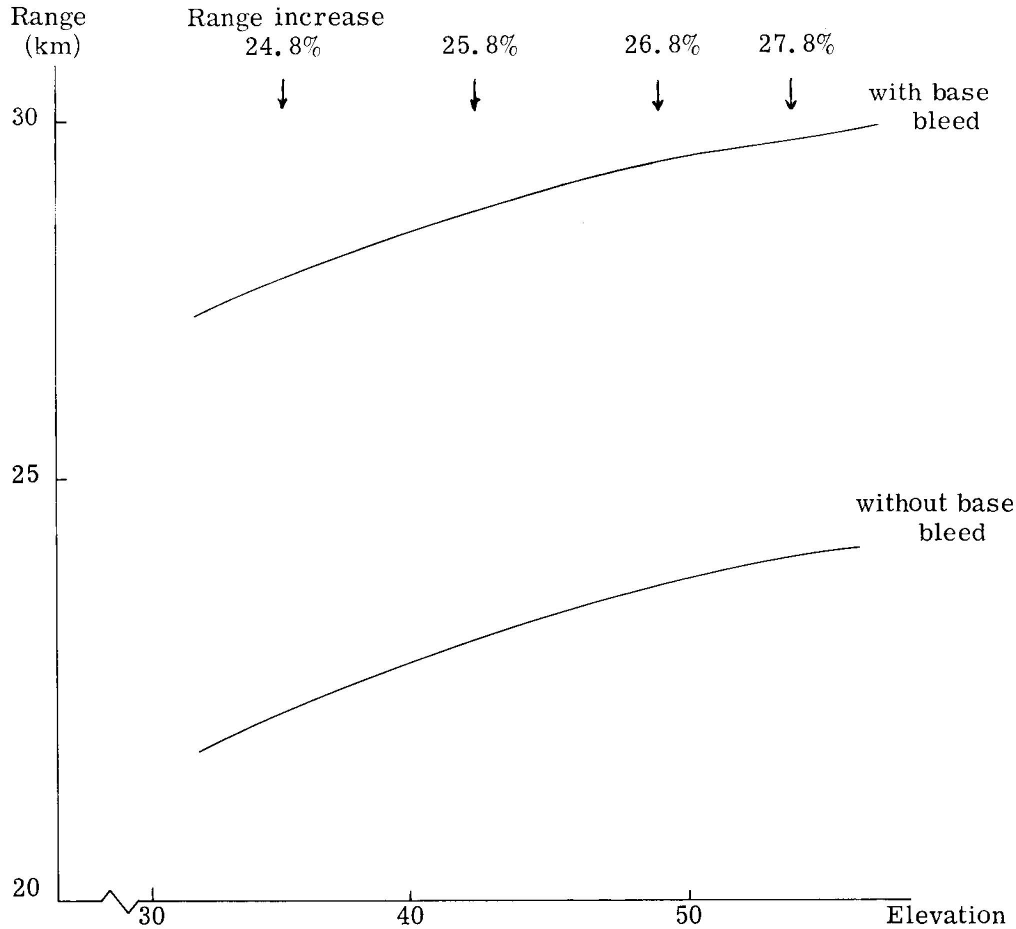


Fig. 4.18 The effect of base bleed on range

A more sophisticated version of base bleed is "external burning" which emits more gas through holes in the periphery of the base. A fuller discussion on practical methods for reducing drag is given in (5).

STABILITY OF A PROJECTILE

Yaw

Except by chance, when a projectile is fired from a gun its axis does not lie exactly along the trajectory, which is the path of its centre of mass. The angle between the axis of the projectile and the tangent to the trajectory is known as the angle of yaw.

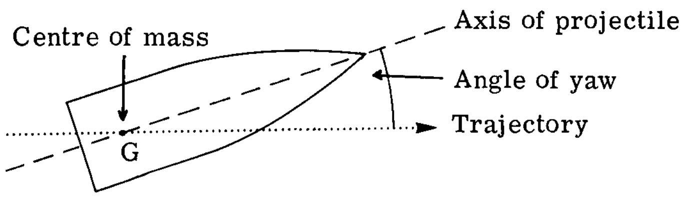


Fig. 4.19 Angle of yaw

Drag, Cross-Wind Force and Over-Turning Moment

The two major aerodynamic forces acting upon the projectile which affect and depend upon yaw are drag and cross-wind force.

Drag retards the motion of the projectile along its trajectory and varies in magnitude with angle of yaw. During normal flight a projectile inevitably yaws slightly, and this produces a relatively small though significant increase in drag. If the projectile yaws to greater angles by design, failure, deflection or encounter with a dense medium (eg. water), the drag will increase dramatically. For example: if a supersonic projectile in air yaws to 90° its drag typically increases a hundredfold.

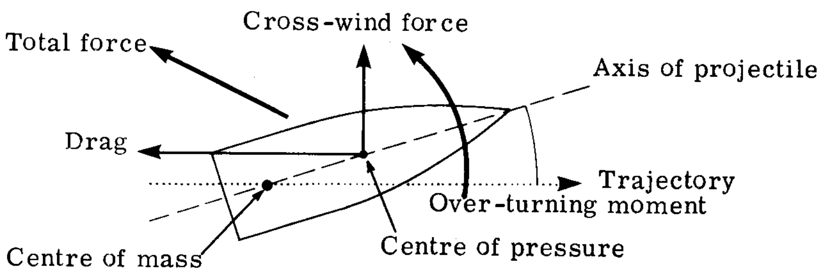


Fig. 4.20 Over-turning moment

Cross-wind force is similar to the lift force generated by a wing. This force acts perpendicular to the trajectory and in the direction of yaw. It is of zero magnitude at zero yaw, and increases with increasing yaw within the limited angles of yaw occurring during normal flight. At extreme angles of yaw the cross-wind force decreases.

These two forces appear to act at approximately the same point on the projectile axis; this point is known as the centre of pressure. As the aerodynamic characteristics of the projectile vary with the constantly changing velocity and attitude of the projectile, so the position of the centre of pressure also varies. For all conventional projectiles the centre of pressure does not coincide with the centre of mass so these combined forces produce an over-turning moment dependent on the angle of yaw. As will be shown later, fin stabilised projectiles are stabilised by the over-turning moment alone; and without the use of spin stabilisation the over-turning moment would cause all other types of conventional projectiles to tumble.

METHODS OF STABILISATION

Fin Stabilisation

The problem of ensuring that the projectile will travel point foremost throughout the flight has been with man since the earliest of times. Arrows and javelins were probably the first elongated projectiles ever used and man soon realised that the best way of stabilising the projectile was to concentrate its mass towards the head. Later tail end fins were added.

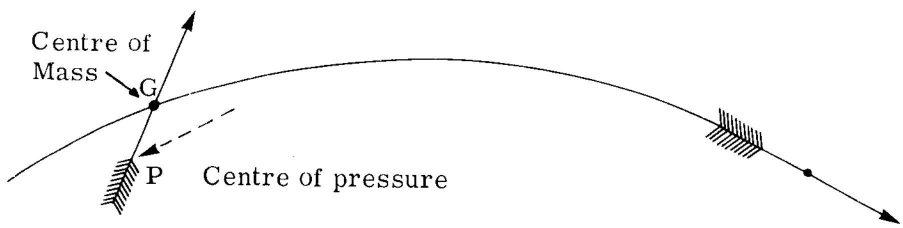


Fig. 4.21 Stability of an arrow

In Fig. 4.21 G is the centre of mass of the arrow. If the arrow yaws and begins to move obliquely, then owing to the larger surface area towards the rear, the resultant air pressure acts through the point P, the centre of pressure, near the tail and so swings the arrow round G and onto the trajectory. As long as the centre of pressure is behind the centre of gravity the projectile will try to fly nose first. This type of stability is called static stability. Stabilisation by fins is used with aircraft bombs, mortar bombs, rockets and Armour Piercing Fin Stabilised Discarding Sabot Shots (APFSDS). The fins are fitted to the rear of the projectile to increase the surface area behind the centre of mass. In this way P is brought behind G and the projectile flies in a state of equilibrium. Fins, of

course, have their disadvantages. They give rise to large wind effects: it is particularly important in the case of a rocket being boosted by the burning of the propellant at the beginning of its flight. The combinations of low level wind and motor thrust causes the rocket to turn into the wind. It is difficult to allow for this adequately in a strong wind, and it becomes a major source of error in free flight rockets which have a long burning time.

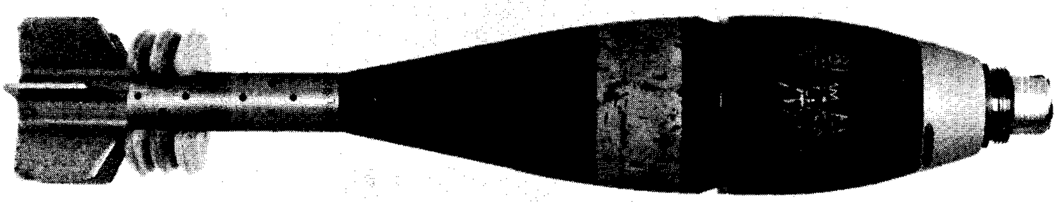
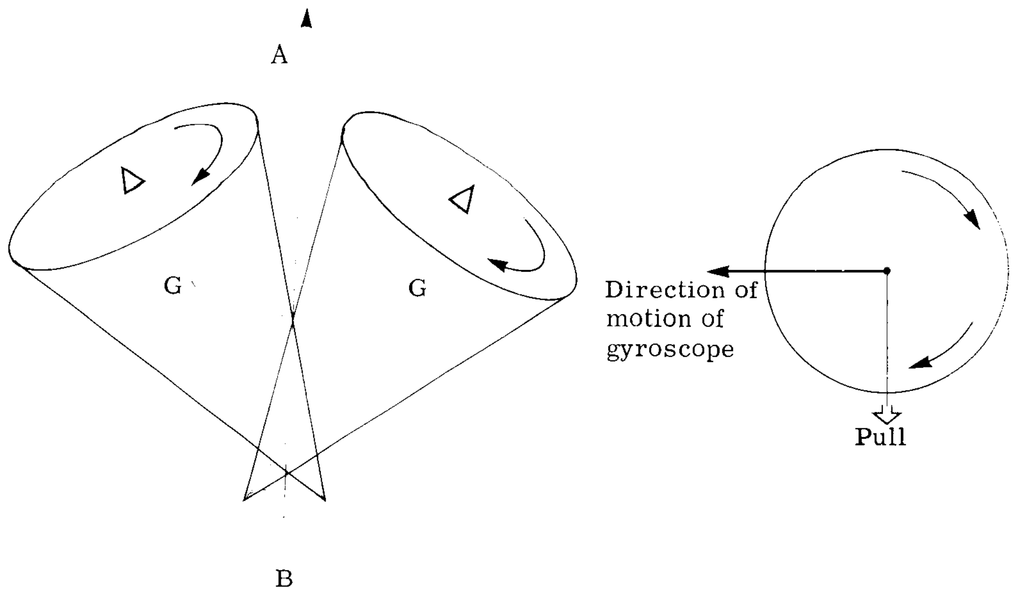


Fig. 4.22 British 81 mm mortar bomb

Spin Stabilisation : Gyroscopes

To withstand the high pressure in the gun, the base of the projectile must be very strong and therefore thick and heavy. We have also seen that a pointed or streamlined nose is required to reduce forebody drag. The combination of the two gives the modern shell its centre of mass near the base, and its centre of pressure near the nose. Unfortunately, as we have seen previously, this results in an unstable projectile. Dynamic stability in a shell is obtained by rapid rotation which produces powerful gyroscopic forces. A gyroscope in its simplest form may be considered to be a heavy top. It is a common experience that a top, if at rest, cannot be made to stand upright, but the moment it is spinning fast enough, not only does it stay in an upright position, but it resists any efforts to pull it out of that position.

Figure 4.23 depicts two views of a spinning top. As it is drawn it would appear that the top has a tendency to fall over. It does not in fact do so and actually moves around with a sort of circular motion, that is, it precesses about the vertical axis AB. Precession may be thought of as a circular yaw about the centre of gravity which takes the shape of a decreasing spiral. If an attempt is made to pull the apex of the top in the direction shown in Fig. 4.23 (b), it will resist the pull. Instead it will try to move in the direction as shown.



(a) Side view of spinning top (b) Elevated view of spinning top

Fig. 4.23 Gyroscopic motion

Another movement observed with a spinning top is called nutation. This is a rotational movement in a small circle which forms a rosette pattern. Relating these movements to a projectile in flight we have the effect illustrated in Fig. 4.24.

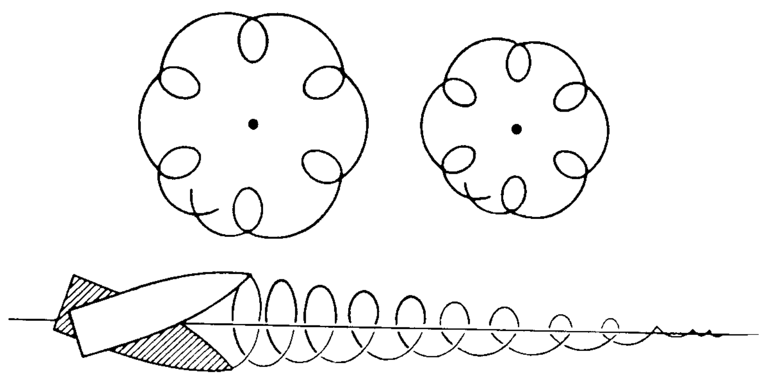


Fig. 4.24 Precession and nutation for a spinning projectile

Both precession and nutation are rapidly damped out by the gyroscopic action of the spin and the projectile is stabilised in flight after a relatively short distance. Now, a top which is short and squat is much easier to spin, and much less affected by random disturbances when spinning, than one which is long and thin. The

criterion of stability for a rotating shell is the same as that for a top, or, in other words, a shell which has a large diameter and which is not too long can be made very stable in flight by the spin imparted to it by the rifling in the barrel. A rocket is usually long and narrow, and to stabilise it by rotation alone is like trying to make a pencil remain upright by spinning it on its point. It can be done, but an extremely high rate of spin is needed. In fact, experiments have shown that once the length exceeds about 7 calibres a projectile can no longer be satisfactorily stabilised by spinning alone, no matter what spin rate is adopted. This means that in the case of rockets, fins provide an easier method of stabilisation; although a very slow spin may be imparted for reasons outlined in the rocket section.

Over and Under Spun Shell

The shell must be made to precess at the correct rate (see Section II) if it is to behave as it should and keep its nose pointing almost along the trajectory. The rate at which the shell precesses depends upon the weight distribution of the shell, its shape, and the position of the centre of gravity; but very important is the rate at which the projectile is spinning relative to the rate at which it is moving forwards and which affects the pressure of the air on it. Referring to the top again, if it is spun at great speed it precesses only slowly, but as the rate of spinning dies away the precession becomes more and more rapid. The same effect occurs in the case of a shell's motion about its centre of gravity. If the shell is spun too rapidly it precesses very slowly with the result that the nose never dips down sufficiently to keep pace with the dipping trajectory and in consequence the shell lands base first. This is the case of the small arms bullet at extreme range and we say in this case that the shell is over-stabilised. Conversely, if the rate of spin is too slow the shell precesses too rapidly and the effect is that the nose dips more quickly than the trajectory. The shell loses considerable range due to the excessive yaw developed and the consequent tumbling. In this case the shell is under-stabilised. A loud whirring noise is characteristic of an under-stabilised shell in flight.

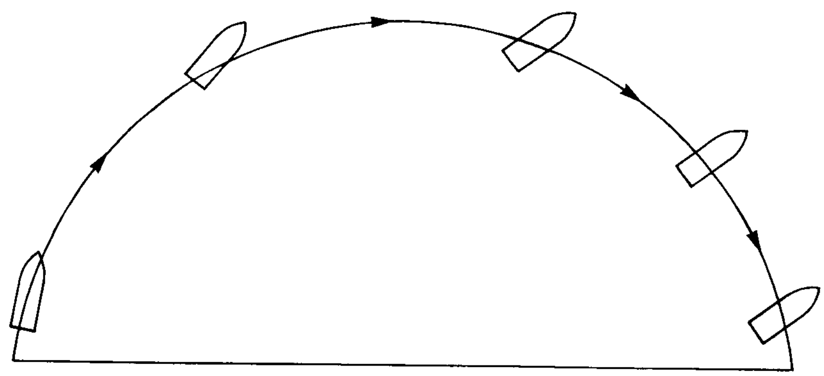


Fig. 4.25 Over-stabilised shell

DRIFT

Definitions

A spun projectile will be found to deviate laterally from the original direction imparted to it by the gun. This lateral deviation is called drift. In practice, this lateral deviation is made up of two parts:

1. Drift due to equilibrium yaw - for spun projectiles.

2. Drift due to rotation of the earth.

Drift Due to Equilibrium Yaw

In our previous analogy of a spinning top with a spinning projectile, we concluded that a spinning projectile is subject to gyroscopic forces. We saw that a pull in the direction indicated below in Fig. 4.26 will result in a movement as shown.

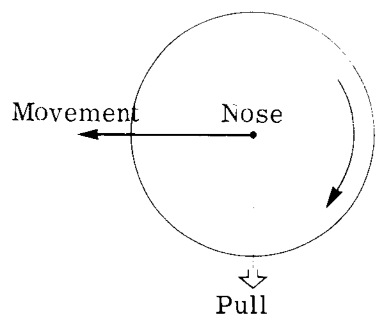


Fig. 4.26 Gyroscopic forces acting on a spinning top

If we now apply the analogy to a shell in flight we see that the net result is that the nose of the shell tends to precess around the trajectory. Soon after being fired, the shell develops some vertical yaw as the trajectory falls. For a shell spinning clockwise the effect is that the nose will move to the right and the base to the left, as in Fig. 4.27.

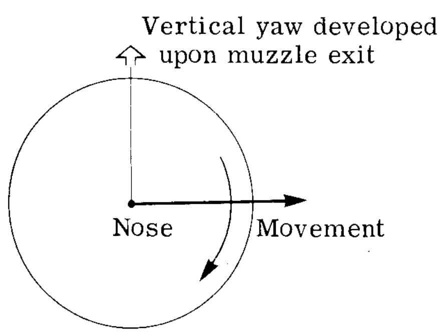


Fig. 4.27 Rear view of spinning projectile : nose movement to the right

Now, the air pressure will tend to increase the yaw to the right, but the gyroscopic effect reacts to this air pressure force, and the nose will tend to move downwards as shown in Fig. 4.28.

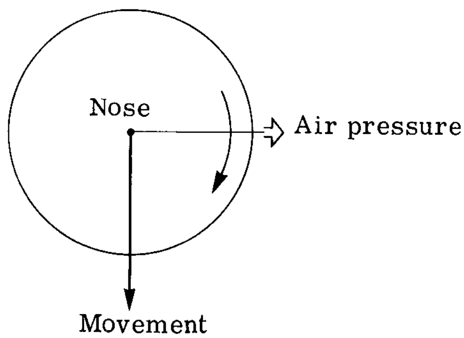


Fig. 4.28 Rear view of spinning projectile : nose movement downwards

Again, air pressure attempts to increase the nose down attitude but the gryroscopic effect moves the nose to the left as shown in Fig. 4.29.

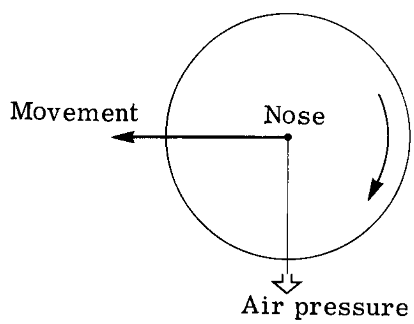


Fig. 4.29 Rear view of spinning projectile : nose movement to the left

By continuing in this way, it is easily seen that the net result is that the nose of the shell tends to precess around the trajectory. However, whilst this is happening, the trajectory is also dipping. The combination of a selected clockwise spin rate, precession, and a dipping trajectory keeps the average yaw more or less constant. The centre of gravity of the shell follows the trajectory and the nose rosettes about the trajectory with an average position off-set to the right. This average yaw to the right is called "Equilibrium Yaw". Similarly, if an anti-clockwise spin is imparted to the shell we have a drift to the left.

The air stream which we can imagine coming towards the shell will therefore, on average, create a high pressure on the left and push the shell to the right. This is the main cause of drift. Corrections can be made to compensate for drift and are usually given in Firing Tables.

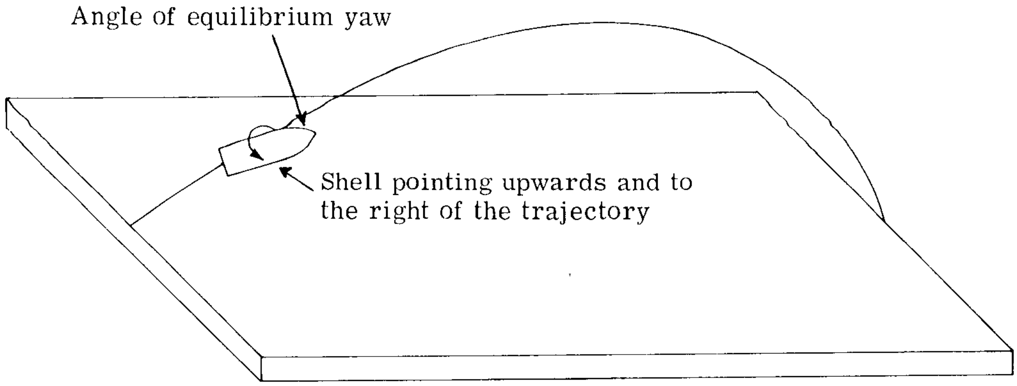


Fig. 4.30 Equilibrium Yaw

At very high angles of departure the nose of the projectile may fail to follow the trajectory at the vertex, with the unfortunate result that the projectile may fall base first, at a reduced range, with a marked drift to the left.

MAGNUS EFFECT

The Magnus force on a shell results from complex aerodynamics. However, the fundamentals of the Magnus effect can be understood by an examination of the effects on a rotating sphere passing through the air.

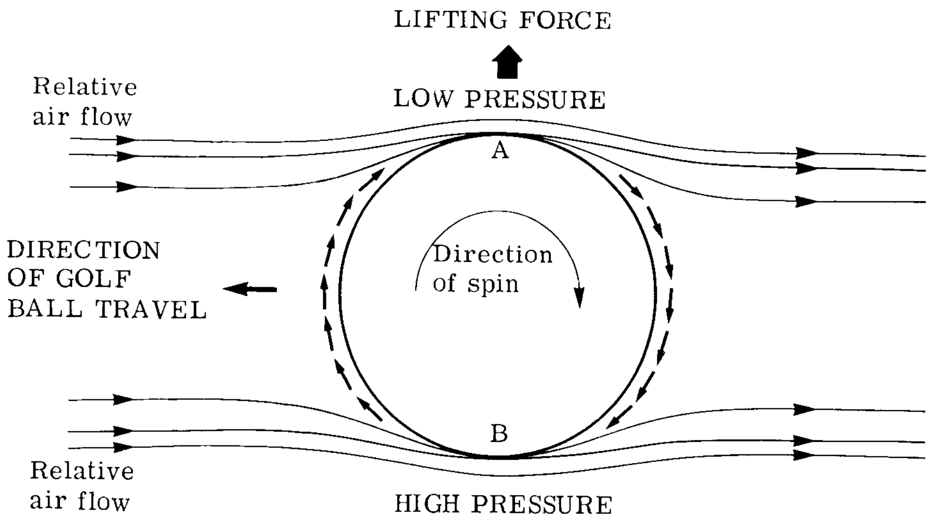


Fig. 4.31 Magnus effect

Figure 4. 31 represents a rotating spherical projectile, for example an undercut golf ball. As the ball spins, it will carry some of the air around with it due to the friction between the air and the surface of the ball; this effect is increased in the case of a golf ball by the addition of indentations on the surface. As there is a meeting of two airstreams travelling in opposite directions an increased pressure results at B. It is this increased pressure which pushes the ball in the direction from B to A, so that a resultant lifting force occurs which increases the range of the ball. At A we do not have this effect because the two airstreams are travelling in the same direction. The same reasoning accounts for the dipping of a top-spun table tennis ball and also for the left drift of a golf ball when hooked.

Essentially the same effect is experienced by a shell. Since, on average, all shells travel with the nose displaced to the right of the trajectory, there is a cross flow of air flow from the left to right of the shell body. In addition, air is carried around by the shell as it spins. It is convenient to interpret Fig. 4. 31 as a section through the shell viewed from the rear. A shell flying with an average equilibrium yaw to the right will experience an average Magnus force upwards, which will tend to increase the range. The Magnus force will also tend to deviate a shell to the left, but the effect is overwhelmed by the presence of gyroscopic forces which cause drift to the right. Since the Magnus force will not necessarily act through the centre of gravity there may also be a Magnus moment in the vertical plane. Although the Magnus force is usually small compared with the equilibrium yaw effect, the Magnus moment contributes to the dynamic stability and is sensitive to the spin rate and angle of yaw. Sometimes the Magnus effect may be large, for example, when an anti-aircraft gun is fired almost vertically. The yaw at the top of the trajectory then becomes much greater and so the Magnus effect becomes more important and the shell may in fact drift to the left.

METEOROLOGICAL CONDITIONS

Properties of the Atmosphere

The atmosphere up to 20 km altitude is basically composed of 78% nitrogen, 21% oxygen with the remaining 1% comprising water, carbon dioxide, hydrogen and other gases. At very high altitudes, separation of the gases occur. For example at about 100 km atomic oxygen separates with the lighter gases diffusing upwards and separating out into further layers.

Pressure, temperature, density and viscosity all vary with altitude. These continual changes in the physical properties of the atmosphere affect the resistance of the air and therefore the range of a projectile. Since the trajectory of small arms, artillery and ballistic missiles have peak altitudes typically of 50 m, 20 km, and 600 km respectively, projectiles fulfilling these roles will experience a significantly different environment during their flight. For example, the density of air decreases as the height above sea-level increases and consequently the range of a missile increases as the trajectory gets higher and higher. If the projectile is fired into the upper atmosphere, where the air resistance is very small, it will go much further. This principle was used by the Germans who launched the 'V2' rockets during the Second World War. Once the 'V2' was about fifty miles

up, there was practically no resistance at all. On the other hand it is the decrease of density with altitude which causes some limitations on the performance of certain aircraft; for example, the helicopter has a maximum operating ceiling due to lack of lift.

The effects of variations in atmospheric conditions depend on the point of the trajectory at which they operate. To predict trajectories the air through which the projectile passes is divided into layers and a weighting factor is determined for each layer. Weighting factors vary not only from layer to layer but also from one trajectory to another. To achieve the greatest accuracy there should be a weighting factor for each layer of each trajectory; in reality this is only practical when calculations are performed on a computer.

It is obvious that the definition of a standard atmosphere is essential to the ballistician as this enables him to undertake stability calculations and range estimations under normalised conditions, rather than for a general situation. It is also desirable to have this standard atmosphere close to some average value of conditions. This requirement is not unique to ballistics, but is equally necessary for aeronautics. It is thus convenient to use the existing standard atmospheres constructed for this purpose. The standard atmosphere most commonly used today is that used by the International Civil Aviation Organisation (or ICAO). Other standard atmospheres include the 1962 and 1976 US standard, the World Meteorological Organisation Standard, the International Organisation for Standardisation, ISO Standard, and the 1942 Standard Ballistic. Figure 4.32 gives an indication of the sort of typical figures given in such atmospheric tables.

It is also essential to define the basic physical properties of a gas:

1. Density (ρ): The density of a gas is its mass per unit volume. The density of the standard atmosphere at sea level, ρ_0 , is 1.23 kg/m^3.

2. Temperature (T): The temperature is a measure of the average kinetic energy of the gas molecules. The Kelvin scale of temperature adopts zero K as absolute zero with Celsius degree intervals. In the American literature the absolute scale of temperature is in degrees Rankine ($^{\circ}$R), and this scale has Farenheit intervals. They are related as follows:

$$^{\circ}R = {}^{\circ}F + 460$$

$$K = \frac{5}{9}\,{}^{\circ}R$$

It will be noticed from Fig. 4.32 that the temperature does not decrease uniformly with altitude.

3. Pressure (P): The pressure of a gas can be related to the density of the gas through the perfect gas equation, which may be written in the form,

$$P = \rho RT$$

where R is a constant for a given gas and has the value 0.287 kJ/kgK in the case

of air. Note that when the temperature remains the same, the pressure increases directly with the density and vice-versa.

Absolute Viscosity

As the pressure on a gas increases the gas becomes more dense. Generally speaking, when a fluid, that is, a liquid or a gas, becomes more dense then it becomes 'stickier', or more viscous. Examples of very viscous fluids are familiar in everyday use, for example, syrup and oil. The important difference between the viscosity in a liquid and a gas is that for a liquid, the viscosity can only increase if it is cooled down, while the viscosity of a gas can vary with the temperature, or the pressure, or both. In fact, the viscosity of a gas rises with an increase in temperature.

Kinematic Viscosity

There is a further important property of fluids used in ballistics and aeronautics called the kinematic viscosity, which is defined to be the absolute viscosity divided by the density ie

$$\text{Kinematic Viscosity} = \frac{\text{Absolute Viscosity}}{\text{Density}}$$

The term viscosity is usually used to mean kinematic viscosity. The increase in viscosity on a hot day is offset by the reduction in overall drag owing to the reduction in air density, so the range is generally increased.

Altitude km	Altitude ft	$\frac{P}{P_o}$	$\frac{\rho}{\rho_o}$	$\frac{T}{T_o}$	$\frac{\nu}{\nu_o}$
Sea level	Sea level	1	1	1	1
5.5	18,000	5×10^{-1}	5.7×10^{-1}	8.76×10^{-1}	1.58
11	36,000	2.25 (-1)	3 (-1)	7.53 (-1)	2.66
15.6	52,000	1 (-1)	1.4 (-1)	7.52 (-1)	5.71
30	100,000	1.05 (-2)	1.3 (-2)	7.89 (-1)	6.2 (1)
47	156,000	1.07 (-3)	1.14 (-3)	9.4 (-1)	8.4 (2)
65	216,000	1.00 (-4)	1.23 (-4)	8.2 (-1)	7 (3)
79	262,000	1.05 (-5)	1.67 (-5)	6.3 (-1)	4 (4)
90	300,000	1.2 (-6)			

Fig. 4.32 Variation of P, ρ, T and ν with altitude

P_0, ρ_0, T_0 and ν_0 refer respectively to the values of the pressure, density, temperature and viscosity at sea level.

Wind Effects

The effect of wind on a particular projectile depends very much on the type of projectile fired. In general a head wind will impede the passage of a shell and a following wind will assist it. A cross wind blowing from left to right will cause the shell to deviate to the right and vice-versa. A shell is not affected quite as much as a finned projectile such as a rocket because its lateral wind resistance is smaller. Wind effects are very important for rocket trajectories, especially for high angles of fire. If the wind is blowing across the trajectory it blows on the fins of the rocket, and as these are at the base, the rocket nose will turn into the wind.

The Speed of Sound in Air (a)

The speed of sound in air can be expressed directly in terms of the temperature of the air as follows:

$$a = \sqrt{\gamma P/\rho}$$

where $P/\rho = RT$ from the gas equation

$$\text{so } a = \sqrt{\gamma RT}$$

where γ is the ratio of specific heats, and for air at atmospheric pressure $\gamma = 1.404$. R is also a constant and was given earlier as 0.287 kJ/kgK for air. If the velocity of the projectile exceeds that of sound the projectile becomes supersonic. From the above relationship, it is evident that the velocity at which transition occurs is directly related to the temperature of the air.

ROTATION OF THE EARTH

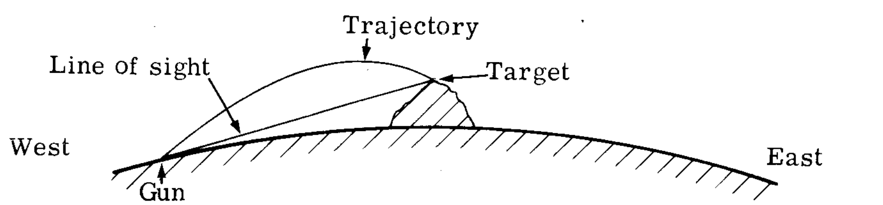
When dealing with long ranges the effects due to rotation of the earth must be taken into account. The rotational speed of the earth's surface at the equator is about 450 m/s, some 1600 km/h. At first sight this appears to be of no consequence as the gun, target and atmosphere are all travelling together, more or less, at the same speed. However, the motion of the earth's surface is not in a straight line, so during the period that a projectile is in flight the target may be carried sideways, up or down by the earth to which it is attached. Fortunately the rotation rate and dimensions of the earth are fixed, so for any gun and target at known positions on the earth's surface it is possible to apply corrections to the gun bearing and elevation to ensure that the projectile meets the target. A rigorous treatment of this drift is beyond the scope of this book. However, it is not difficult to get an adequate grasp of the principles involved.

As the earth's rate of rotation is constant, the angle through which the earth rotates during a projectile's flight is dependent upon the time of flight, and the extent of the resultant drift is magnified by the range of the trajectory. Consequently, drift of this form is of significance only to trajectories featuring both long range and long time of flight. Of course long ranges imply long flight times, so very long ranging projectiles, notably ballistic missiles, are severely affected by this drift. The drift of artillery shells is much less. At a range of 20 km the drift may be 100 m, depending on the gun's geographic position, the bearing to the target and the time of flight. Over ranges of less than 5 km the drift is generally much less than the shot to shot variations of point of impact, whilst for small arms the drift due to rotation of the earth is so small that it is always ignored.

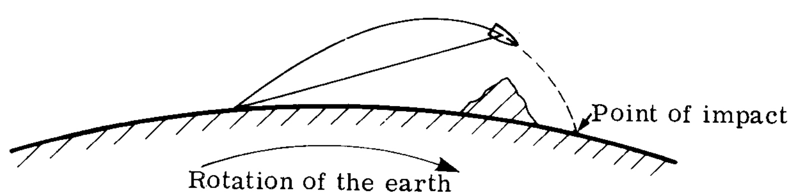
There are three forms of drift that can be distinguished; the first two primarily affect the range of the projectile, and the third mainly affects the bearing to the target.

The first form is most apparent when a projectile is fired directly upwards at the equator. During its period of flight the earth rotates so that the projectile lands at a point west of the launching point.

The second form of drift is observed for trajectories between points at the same latitude. When firing east, in the same direction as the rotation of the earth, the earth effectively rotates down and away from the trajectory so the projectile passes over the target and strikes the earth some distance beyond. Conversely, firing west results in the projectile falling short of the target. Whether firing east or west, this form of drift is always eastward. In addition, this same effect causes a very slight sideways drift of trajectories at moderate latitudes as the projectile follows a 'great circle' route while the target rotates equilaterally.



Required trajectory if rotation of the earth is ignored



Subsequent position of projectile above target at required range

Fig. 4. 33 Extension of range when firing eastward at low elevation

These two forms of drift are greatest at the equator and lessen towards the poles. They have an opposing effect upon range; the first dominates drift at very high elevations of the gun, and the somewhat greater drift due to the latter form dominates lower elevation trajectories. At a certain elevation these two drifts are equal and opposite, so they have no overall effect upon the range; this elevation is generally about 60°.

For the final form of drift consider a gun firing across the north pole. During the time of flight the earth rotates anti-clockwise beneath the trajectory, so causing the projectile to land to the right of the point of aim. This drift to the right occurs throughout the northern hemisphere, though lessening towards the equator and absent at the equator itself. The effect is reversed in the southern hemisphere, so causing a drift to the left.

These three forms of drift have a superimposed effect. For guns the required adjustments to bearing and elevation to correct for the drift are tabulated in firing tables to suit all possible trajectories.

ROCKETS

Velocity Considerations

In this section we will look at the basic characteristics of a free flight rocket; that is, a rocket which has all its guidance given to it by the launcher. A rocket consists of many parts. The head is designed as a shell, while the rear part of a rocket is normally taken up by the motor, fuel and combustion chamber. The chamber is analogous to the chamber of a gun in which the propellant is burned. By means of a nozzle system the products of combustion are ejected at high speed from the rear of the motor. These gases exhaust with a certain momentum, and according to Newton's Second Law of Motion, the rate of change of momentum of the gas leaving the rocket represents a force, and, according to his third law, this force is reacted against by a thrust in the opposite direction. It is this thrust that drives the rocket forward.

Rockets may have either liquid or solid fuels. Generally speaking liquid fuel rockets have a high energy content, while the solid fuel rockets are relatively simple to produce and easy to handle, but have a relatively low energy output. The relative merits of liquid and solid propellants are discussed in (3). In a vacuum the maximum velocity of a rocket depends on the speed of the emerging gases and on the ratio of the weight of the rocket before firing to the weight when the propellant is all burnt. This ratio is called the "mass-ratio". The acceleration at any moment depends on the rate at which the fuel is being consumed, the speed of the emerging gases, and on the remaining weight of the rocket. The maximum speed will be reached just when the fuel is completely burnt. When the rocket is fired in the atmosphere these statements are only approximately true. All the external effects must be considered as was the case with shells. In general the range of a rocket may be increased by choosing the composition of the fuel so that the speed of the emerging gases, or exhaust velocity, is as large as possible, and by designing the projectile to make the mass-ratio as large as possible. There are

however practical problems associated with the design of rockets; a good account being given in (3).

Rocket Accuracy and Stability

The principal disadvantage of free flight rockets is their relatively poor accuracy and consistency compared with guns. The main sources of error are due to wind effects, thrust misalignment and velocity at all burnt. As the rocket is moving slowly when it leaves the launcher, surface side wind effects on the stabilising fins can be large and the rocket tends to turn into the wind. Wind corrections are therefore applied at the latest possible moment before firing to reduce this error. The thrust from the rocket motor must act along the axis of the rocket. If it is offset it will tend to act like a rudder and deflect the rocket off course. This misalignment can be partially overcome by spinning the rocket slowly. A small change in the rate of burning of the rocket propellant can make a significant difference to the velocity of the rocket at all burnt which in turn will affect the final range achieved. Errors can also arise if the rocket centre of gravity is not on the nozzle axis.

There are three main methods of achieving stabilisation of free flight rockets: they are fin stabilisation, spin stabilisation and a combination of fin and spin stabilisation. Large fins are required to minimise the flight deviations arising from launch and thrust misalignment, but small fins are necessary to reduce the deviations arising from surface cross winds. The optimum size of fin is chosen to achieve the least overall dispersion. Slow spin is imparted during launch and the resultant motion of the free flight rocket aids the rocket in maintaining its stability about its vertical, longitudinal and lateral axes. Most free flight rockets achieve stability by spin and fin stabilisation. Two principal methods used are a slow rate of spin imparted to the rocket while still on the launcher and auxiliary rocket motors which are fired to spin the rocket immediately after it is launched.

This ends the non-mathematical treatment of external ballistics. For those readers who have no wish to go deeper, it is suggested that they miss out Section II of this chapter and move onto Chapter 5. Anyone who wishes to study the subject in more depth should read on.

SELF TEST QUESTIONS

QUESTION 1 For a projectile fired in-vacuo, what angle of projection gives the maximum range?

Answer ....

QUESTION 2 If a projectile generates a conical shock wave with an angle of 30°, estimate the velocity of the projectile.

Answer ....

....

QUESTION 3 What is meant by "base-drag" acting on a projectile in flight?

Answer ....

....

....

....

QUESTION 4 Give three practical methods of reducing the effect of base-drag.

Answer ....

....

....

QUESTION 5 Why are muzzle velocities in the region of the velocity of sound in air avoided by the designer of projectiles?

Answer ....

....

QUESTION 6 If an unspun projectile acquires a small yaw, what three factors will result?

Answer ....

....

....

QUESTION 7 Why are rockets not stabilised by spin alone?

Answer ....

....

....

QUESTION 8 What are the four possible types of drift that may act on a projectile?

Answer ....

....

....

....

QUESTION 9 Why is it necessary to consider the physical properties of the atmosphere?

Answer ....

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....

QUESTION 10 For what type of projectile is the rotation of the earth an important consideration?

Answer ....

QUESTION 11 What action drives a free flight rocket forward?

Answer ....

....

....

ANSWERS ON PAGE 203

External Ballistics — Part II

CONCEPT

In this section of external ballistics an attempt is made to provide the reader with a deeper and more theoretical study of some of the basic concepts presented in Section I of this chapter. It is now necessary to refine our views by examining in more detail the various aspects such as trajectories, fluid motion and aerodynamics. A study of these areas of ballistics is vital in order to gain a complete understanding of the in-flight behaviour of projectiles.

TRAJECTORIES

Assumptions

A large part of external ballistics consists of evaluating trajectories. There are numerous forms of trajectory modelling involving different basic assumptions and having different complexities of solution. The majority of trajectory modelling undertaken nowadays uses numerical integration by computer and this has had the effect of rendering obsolete many of the analytical and semi-empirical techniques. In this section we shall assume that the projectile behaves as a point mass and only consider the influence of gravity, drag and wind effects. Once we leave point mass trajectories, other considerations such as the angle of yaw, aerodynamic moments, etc, have to be made; and a treatment of these effects in trajectory modelling is beyond the scope of this book. A full treatment may be found in (3).

Parabolic Motion in Vacuo

Considering the notation of Fig. 4.34 overleaf, we may write down the equations of motion in the horizontal (x-direction) and vertical (y-direction) by using Newton's second law of motion, as follows:

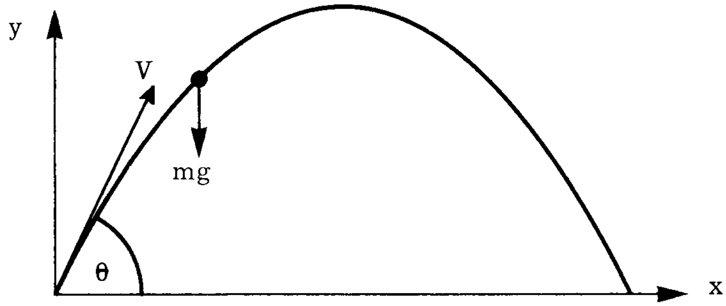

Fig. 4.34 Parabolic motion in vacuo

$$m\frac{d^2x}{dt^2} = m\ddot{x} = 0$$

$$m\frac{d^2y}{dt^2} = m\ddot{y} = -mg$$

where $g = 9.81\ m/s^2$ is the acceleration due to gravity. Integrating these equations with respect to time, and using the initial conditions that $\dot{x} = V\cos\theta$, $x = 0$, $\dot{y} = V\sin\theta$, $y = 0$ at time $t = 0$, the instant of projection, we have:

$$\frac{dx}{dt} = \dot{x} = V\cos\theta \quad \frac{dy}{dt} = \dot{y} = V\sin\theta - gt$$

$$x = V\cos\theta t \qquad y = V\sin\theta t - \tfrac{1}{2}gt^2$$

Putting
$$t = x/V\cos\theta \tag{1}$$

in the expression for y results in the equation connecting y and x:

$$y = x\tan\theta - \tfrac{1}{2}g\frac{x^2}{V^2\cos^2\theta} \tag{2}$$

which is the equation of a parabola.

Horizontal Range and Time of Flight

Referring to (1.2), we see that $y = 0$ when $x = 0$ or when

$$x = \frac{2V^2}{g}\sin\theta\cos\theta$$

ie. $$x = \frac{V^2}{g} \sin 2\,\theta \quad \text{which is the horizontal range.}$$

The maximum horizontal range is obtained when $\theta = 45^{\circ}$ when the range is V^2/g.

When $$x = \frac{V^2}{g} \sin 2\,\theta, \quad \text{we have from equation (1)}$$

that $$t = \frac{2V}{g} \sin\theta \qquad \text{giving the time of flight.}$$

Computation of Trajectories by Computer

The methods of calculating trajectories using tables have been largely superseded by the use of computers which integrate the x and z differential equations of the trajectory illustrated in Fig. 4. 35. These are, in aerodynamic notation:

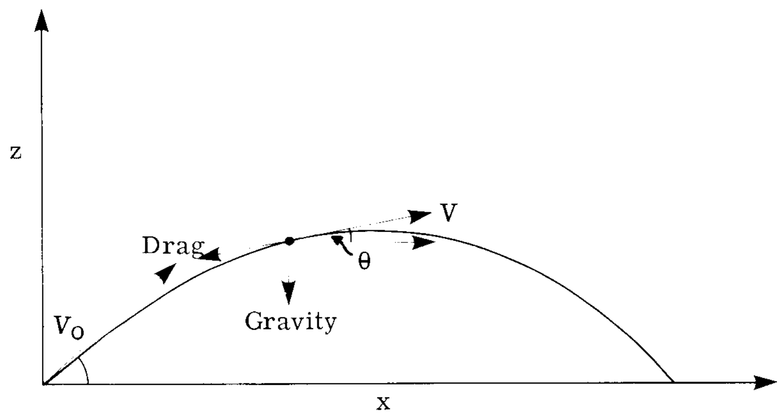

Fig. 4. 35 The effect of gravity and drag acting on a projectile

$$m\ddot{x} = \tfrac{1}{2}\rho V^2 A C_D \dot{x}/V$$

$$m\ddot{z} = -\tfrac{1}{2}\rho V^2 A C_D \frac{\dot{z}}{V} - mg$$

where $V^2 = \dot{x}^2 + \dot{z}^2$, with initial conditions $x = 0$, $z = 0$, $\dot{x} = V_o \cos\theta$, $\dot{z} = V_o \sin\theta$ when $t = 0$.

The equations above are integrated by a step-by-step process. They are first re-written as a set of first order differential equations by introducing the new variables $V_X = \dot{x}$ and $V_Z = \dot{z}$. The first order equations may then be integrated by a fifth order Runge-Kutta technique. A variety of instructions can be keyed in and a graphical output obtained.

Rocket Trajectories

So far we have been considering trajectories of projectiles such as cannon balls, mortar bombs and shells which are not powered in flight. Let us now consider the trajectories of rocket propelled projectiles which are powered. The motion of a rocket may be divided into three phases: 1. the launch phase, 2. the boost phase and 3. the ballistic phase. In the launch phase a launch tube is used to direct the initial motion of the rocket. Launch dynamics is a complete study in itself and a good account of the subject may be found in (3). Once the rocket has accelerated to its maximum velocity we have an "all-burnt" situation and the motion enters the ballistic phase. This part of the trajectory is similar to that of a shell and will not be discussed further. The period of greatest concern for rocket accuracy is the boost phase between launch and all-burnt, during which time, under the influence of rocket thrust, the motor is sensitive to initial disturbances arising from the launch: two of these are wind and thrust misalignment. A full theoretical account of rocket motion may be found in Rankin (6). In the simplest treatment of the rocket trajectory it is assumed that launch is perfect and the rocket moves under the influence of gravity, axial thrust and drag forces only. We can therefore assume that the rocket behaves as a point mass. We ignore all the forces and couples acting on the projectile as these will, on average, often cancel each other out. The point mass assumption can be used in the preparation of range tables and ballistic data.

Rocket Thrust

The action of a rocket motor depends on conservation of linear momentum. Let us consider the situation depicted in Fig. 4.36. The total momentum at time t is equal to the total momentum at time t + δt.

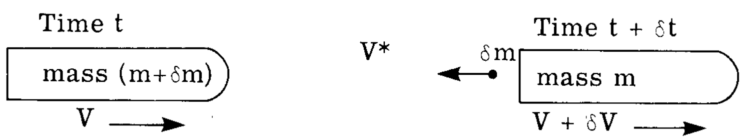

Fig. 4.36 Conservation of linear momentum

In the short time interval δt a particle of mass δm is ejected with a velocity V^* relative to the rocket. Equating the momentum before and after the ejection of this particle

$$(m + \delta m)V = m(V + \delta V) + \delta m(V + \delta V - V^*).$$

Cancelling terms mV and δmV and disregarding $\delta m\delta V$ as it is second order we obtain

$$m\delta V = mV^*.$$

Now $\delta m = -\frac{dm}{dt}\delta t$ since $\frac{dm}{dt}$ is the rate of increase of mass and is negative since m is in fact decreasing with time. Dividing by δt and letting $\delta t \to 0$ we obtain the differential equation of motion as

$$m\frac{dV}{dt} = -\frac{dm}{dt}V^*.$$

V^* is known as the exhaust velocity of the rocket and may quite often be taken as a constant. The expression $T = -\frac{dm}{dt}V^*$ may be regarded as the thrust of the rocket. In this case the equation of motion can be integrated to give

$$\int_{V_o}^{V} dV = -V^* \int_{m_o}^{m} \frac{dm}{m}$$

or

$$V - V_o = V^* \log_e \frac{m_o}{m}$$

where V_o is the initial speed of the rocket and m_o is the initial mass. In the above calculation air resistance has been neglected. The thrust, $T = -\frac{dm}{dt}V^*$ is quite often very large compared with the drag and gravity so that this omission is justified. The equation of motion neglecting gravity but including drag is

$$m\frac{dv}{dt} = T - D$$

where T is the thrust of the rocket and D the drag. In practice V^* is the effective exhaust velocity since the thrust of a rocket is composed of two parts: they are a mass loss term and a pressure term. Here the thrust is given by

$$\begin{aligned} T &= -\frac{dm}{dt}V'\,(P_e - P_a)A_e \\ &= -\frac{dm}{dt}V^* \end{aligned}$$

where V' is the true exhaust velocity and V^* the effective exhaust velocity. A_e is exit area, P_e exit pressure and P_a atmospheric pressure. Additionally the quantity V^*/g which has the dimension seconds is called the specific impulse. Specific impulse depends on the propellant: for the "V2" rocket it was 205s, for the "Redstone" missile 235s and for the "Atlas" missile 250s. Figure 4. 37 shows some exhaust velocities and specific impulses for various types of propellant:

	Specific Impulse (sec)	Exhaust Velocity (m/sec)
75% ethyl alcohol and liquid oxygen	279	2733
Liquid hydrogen and liquid oxygen	391	3837
Fluorine and hydrogen	410	4024
RIP and hydrogen peroxide 95%	273	2680

Fig. 4.37 Some exhaust velocities and specific impulses for various types of propellant

It is also important that the mass ratio m_0/m (mass at launch over mass at burn-out) is high; for a "V2" rocket the mass ratio was 2.8, whilst a modern rocket has a typical value of about 5.

Rocket Drag

The drag whilst the motor is burning is different from the free flight drag and so adjustments are made to account for base drag. The situation is thus taken to be:

Drag during burning = Resistance of shell of same calibre and head shape
+ skin friction of rocket - skin friction of shell
- base drag over nozzle area of rocket.

For conventional artillery rockets the exhaust plume removes the base drag contribution during boost, so

Drag after all-burnt = Same as above but omitting base drag over nozzle area term.

The skin friction drag is taken as $\frac{1}{2}\rho V^2 S C_F$ where S is the surface area of the rocket and C_F is the skin friction drag coefficient. Usually turbulent flow is assumed for which

$$C_F = 0.455(\log_{10} Re)^{-2.58}$$

where $Re = Vl/\nu$ and l is a typical dimension of the rocket body. The skin friction acting on fins is usually considered separately, l being their streamwise chord length. Recently, results of wind-tunnel tests have been used to calculate drag on a variety of rocket propelled projectiles.

Equations for Rocket Trajectories

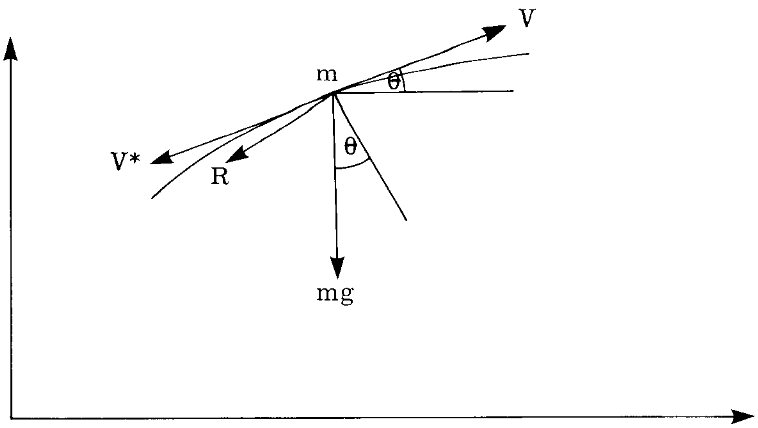

Fig. 4.38 Rocket trajectories

The equations of motion along and at right angles to the trajectory may be written as

$$m \frac{dV}{dt} = - \dot{m}V^* - R - mg \sin \theta$$

$$mv \frac{d\theta}{dt} = - mg \cos \theta$$

If $- \dot{m}V^*$ is large compared with R and mg sin θ we may integrate the first equation to give $V = V^* \log_e (\frac{m_o}{m})$ at all burnt.

The acceleration $\frac{dV}{dt}$ is often uniform so that along the trajectory $V = ft$ and the distance $x = \frac{1}{2}ft^2$ where f is the uniform acceleration. θ may then be found by putting $V = ft$ into the second equation and integrating to obtain

$$\theta = \theta_o - \frac{g \cos \theta_o}{f} \log_e \left(\frac{t}{t_o}\right)$$

Here cos θ has been considered to be constant and equal to $\cos \theta_o$. t_o is the instant the rocket leaves the launcher with velocity ft_o. If however $-\dot{m}V^*$ is not lareg compared with drag etc. and wind terms are important we have the following equations of motion:

(a) During burning

$$m\dot{\underline{v}} = (- \tfrac{1}{2}\rho A |\underline{v}-\underline{w}|^2 \tilde{C}_D - u_e \frac{dm}{dt}) \frac{(\underline{v} - \underline{w})}{|\underline{v} - \underline{w}|} - mg\hat{j}$$

where $\underline{v}$ = velocity = $u_x\hat{i} + u_y\hat{j} + u_z\hat{k}$ (N.B. y is now vertical coordinate).

$\underline{w}$ = wind velocity = $w_x i + w_z k$ (ie has no vertical component).

$\frac{dm}{dt}$ = rate of loss of mass

u_e = effective exhaust velocity

V^* = "specific impulse" x g

$\tilde{C}_D$ = drag - (base drag)

(b) After all burnt

$$m\dot{\underline{v}} = \tfrac{1}{2}\rho A\,|\underline{v}-\underline{w}|^2\, C_D\, \frac{(\underline{v} - \underline{w})}{|\underline{v} - \underline{w}|} - mg\hat{j}$$

where C_D is the drag plus base drag.

A computer program may be written to solve the above equations.

FLUID MOTION

Fluids

The word fluid is used to describe both liquids and gases. Of particular interest to ballisticians is the resistance of a projectile to motion caused by the viscosity or friction of the air. We have already defined the absolute viscosity (μ) of a gas as a measure of the flow resistance of the gas, and the kinematic viscosity (ν) as the ratio of absolute viscosity to density. We now define another important quantity called the Reynold's Number (Re):

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}}$$

The inertial force acting on a typical fluid particle is measured by the product of its mass and its acceleration. Now the mass per unit volume of the fluid is, by definition, the density ρ, while the volume of the particle is proportional to the third power of the length scale. Hence the mass is proportional to ρL^3 or mass is proportional to L^3.

The mean acceleration is the change of velocity divided by the time in which this change occurs. The changes in velocity as the fluid accelerates and decelerates in the neighbourhood of the body are of the same order of magnitude as the speed, V, itself. The time in which this change occurs is of the same order as the time for an average fluid particle to travel the length L of the body at the speed V;

this time is L/V. Accordingly the acceleration is proportional to V divided by L/V, that is

$$\text{acceleration} \propto V^2/L.$$

Therefore the inertia force = mass x acceleration

$$\sim \rho L^3 V^2 / L$$

$$= \rho L^2 V^2$$

where ~ here means both "proportional to" and "of the same order as".

The viscous forces are determined by the viscous shear stress acting on the body and the surface area (L^2) over which it acts. The viscous stress is proportional to viscosity μ and to the rate of change of speed with distance, that is V/L. Therefore

$$\text{Viscous Forces} \sim V \mu L$$

We can now define Reynold's number more precisely as:

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} \sim \frac{\rho L^2 V^2}{V \mu L} = \frac{\rho L V}{\mu} = \frac{L V}{\nu}$$

where V is velocity (m/s)

L is length (m)

μ is absolute viscosity (kg/m sec)

ν is kinematic viscosity (m^2/s)

We are now in a better position to define the drag coefficient.

The Drag Coefficient

The drag force D acting on a projectile in flight is usually written in the form

$$D = \tfrac{1}{2} \rho V^2 A C_D$$

where ρ = air density (kg/m^3)

V = velocity of projectile (m/s)

A = reference area of projectile (m^2)

C_D = drag coefficient

C_D is dimensionless and is a function of two further dimensionless quantities:

$$\text{Reynold's number, } Re = Vd/\nu$$

and

$$\text{Mach number, } M = V/a$$

where d is a typical dimension and a the velocity of sound in air.

Reynold's number represents the effect of viscosity and Mach number the effects of compressibility. In the SI system of units:

1. The density of dry air at $15^{o}C$ and a pressure of one atmosphere (1.013×10^5 N/m^2) is 1.225 kg/m^3.

2. The viscosity, μ, of dry air is 1.78×10^{-5} kg/m sec and the kinematic viscosity $\nu = \mu/\rho = 1.45 \times 10^{-5}$ m^2/s.

3. The velocity of sound in air under these conditions is 340.6 m/s.

In addition to the components described above, C_D depends on the shape of the projectile and the yaw angle. We have already seen in Section 1 the relationship between the drag coefficient and Mach number. We will consider the variation of C_D with Reynold's number, Re, later.

Dynamic Similarity

If the Reynold's number is the same for similarly shaped objects in different fluids, then the flows about the objects are equivalent. This equivalence is called dynamic similarity. Data provided by Shapiro (7) shows exactly how the principle of dynamic similarity makes it possible to predict the performance of full-scale projectiles (or aircraft) from relatively convenient and inexpensive wind tunnel tests of scale models.

	Length L	Speed V	Fluid	Density ρ	Viscosity μ	Re $\frac{\rho VL}{\mu}$	Drag Force	Drag Coefficient
Helium-filled Balloon	100	5.4	AIR	0.0012	0.018	36	0.0067	0.000019
Plastic Ball	0.60	55	WATER	1.00	0.89	37	0.020	0.000018

Fig. 4.39 Dynamic similarity

For comparison a small plastic ball with a diameter of 0.60 cm was dropped into water, while a large helium-filled balloon with a diameter of 100 cm was allowed to rise in air. For the small ball the Reynold's number was calculated to be 37 and for the gas balloon 36. Thus, for these two particular experiments the Reynold's number differs very slightly. So while these experiments, one in gas,

one in a liquid, one large, one small, one ascending and the other descending, appear very different to the eye, they are in fact dynamically similar. The important practical implication of this is that the condition of dynamic similarity enables us to predict the drag of the helium balloon in air, by making a different experiment in which we measure the drag of the plastic ball in water. Dynamic similarity is the key to how we are able to predict, from the measurements made in the wind tunnel on a model projectile such as a rocket, the forces which the real rocket would actually experience.

The Relationship between Drag Coefficient and Reynold's Number

It may be shown that when experiments are carried out on geometrically similar shapes at the same Reynold's number, the drag forces on the objects, while perhaps very different in magnitude, are related in a very special way. Referring to the experiment with the ball and balloon we again see that the drag coefficients are very similar, as shown. This leads to the law of dynamic similarity: equality of Reynold's number implies equality of drag coefficient. A typical graph of the variation of the drag coefficient with Reynold's number is given below.

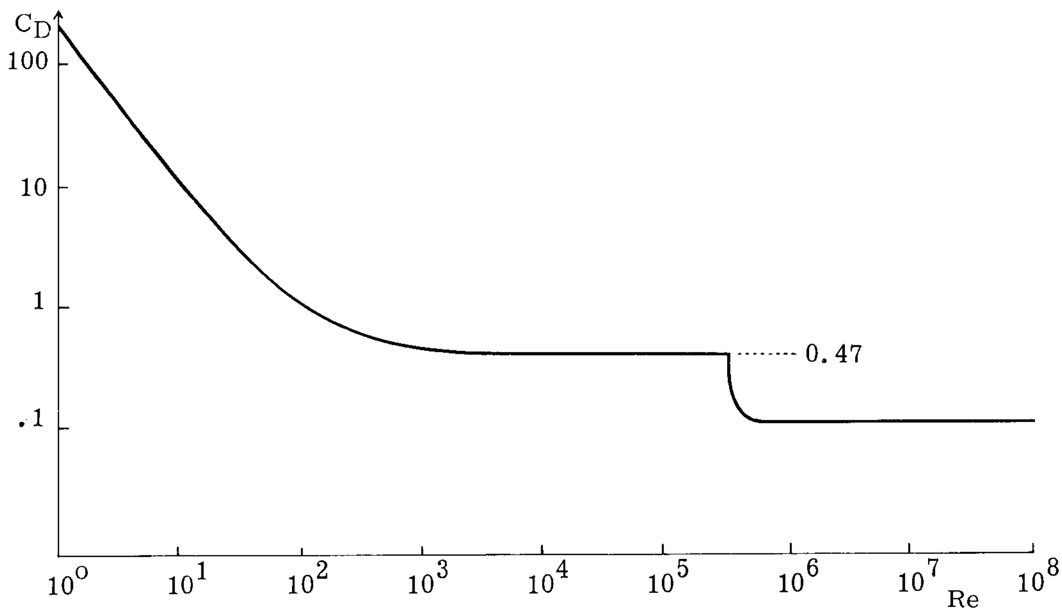

Fig. 4.40 The relationship between C_D and Re

Laminar and Turbulent Flow

Laminar flow in a fluid is easily seen in the smooth streamlined motion observed in freely flowing thick oil. It is difficult to make the oil turbulent or chaotic by mixing since any irregularities in the flow are quickly damped out by strong viscous action. In laminar flow the fluid particles move downstream in smooth and regular trajectories, without appreciable mixing between different layers of fluid.

Such a laminar flow is termed steady. In a turbulent flow there is superimposed on the average downstream motion of the fluid an irregular and seemingly random motion. In the mixing process any slow-moving fluid is invigorated and speeded up by fast-moving fluid with which it mixes, while the latter in turn tends to be slowed down.

We can determine whether a fluid motion will be laminar or turbulent by considering the size of Reynold's number.

If Re is very large, say of the order 10^7, we know that the inertia forces will dominate, and if Re is very small, say of the order 10^2, viscous forces will dominate. In general,

If Re $< 10^4$, the flow is orderly and steady and this is defined as "laminar flow".

and if Re $> 10^6$, then the flow is very energetic and this is called "turbulent flow".

Viscous Boundary Layer

In all fluids the particles immediately in contact with solid boundaries do not slip relative to the boundary. Therefore, while most of the air may rush past a projectile at high speed, the air at the surface of the projectile does not move at all relative to the projectile. Between this film of relatively motionless air next to the projectile and the fast moving main body of air at some small distance from the projectile, there is a region of flow called the boundary layer.

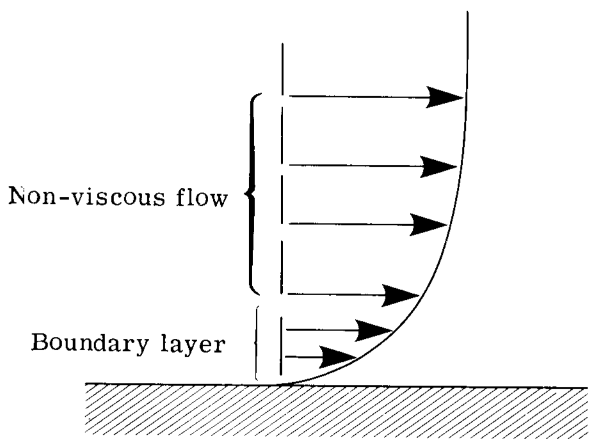

Fig. 4.41 Viscous boundary layer

In this layer the speed changes from zero at the surface, to a velocity virtually unimpeded by viscous forces at the outside edge of the layer. The formation of the boundary layer is initiated by the motionless layer of fluid at the surface of the projectile. This motionless layer exerts a viscous drag on the layer of fluid next

to it, thus gradually slowing down that second layer. This effect continues from layer to layer. In this way the decrease of velocity at the surface is diffused by viscosity outward through the fluid, and more and more fluid becomes caught up in the viscous boundary layer. The continual thickening of the boundary layer is shown in general for a fluid stream travelling from left to right past a flat surface.

The arrows represent the thickness of the layer at successive instants of time. The amount by which the boundary layer grows in a certain time or in a certain distance depends on such things as the speed, the viscosity, and the density. In general, as the Reynold's number increases, the boundary layer grows less rapidly. It should be noted however, that no matter how large the Reynold's number becomes the viscous boundary layer can never totally disappear.

As Re decreases, the boundary layer becomes thicker and thicker. At extremely low Re, the viscous region extends very far from the body, and occupies practically the entire region of flow. It is meaningless then to even speak of a boundary layer.

In general the boundary layer grows in thickness from the nose of a projectile, and has a different law of growth according to the type of flow. For a laminar boundary layer

$$\frac{\delta_{lam}}{x} = 5.5\ Re_x^{-0.5}$$

where δ_{lam} is the thickness at a distance x from the nose or leading edge and Re_x is the Reynold's number at the same point. For a turbulent boundary layer the growth is different and is given by

$$\frac{\delta_{turb}}{x} = 0.22\ Re_x^{-0.2}$$

For ballistic purposes we are mostly concerned with turbulent flow and turbulent boundary layers. The state of the boundary layer indicates:

1. The effect that protuberances or roughness have had on the skin friction.

2. The sensitivity of the flow field to body curvature.

3. The extent of the cavity in the base region of the body.

Two ballistic examples are:

1. For a 7.62 mm calibre, 28 mm long bullet, having a muzzle velocity of 840 m/s,

$$Re = \frac{840 \times 0.028}{1.45 \times 10^{-5}} = 1.62 \times 10^{6}$$

where the kinematic viscosity of air is taken at sea level. A turbulent boundary layer is indicated with a boundary layer thickness of 0.35 mm at the bullet base.

2. For a 9 m long rocket travelling at 600 m/s at an altitude of 5.6 km

$$Re = \frac{600 \times 9}{1.45 \times 1.58 \times 10^{-5}}$$

$$= 2.4 \times 10^{8}$$

In example 2. we have used a formula relating the kinematic viscosity at sea level and some other height. The Reynold's number found represents a well established turbulent boundary layer with a thickness at the rocket base of 42 mm.

At high Re there is separation of the airflow while at low Re there is not. At first sight it may seem natural to think that a turbulent boundary layer would produce a larger drag force. However this is not necessarily true, because a turbulent boundary layer remains attached longer and produces a smaller wake. This effect is most readily observed when the flight of a golf ball is considered. The boundary layer becomes turbulent for a roughened sphere earlier than it does for a smooth sphere and consequently a smaller wake is produced due to the boundary layer separation.

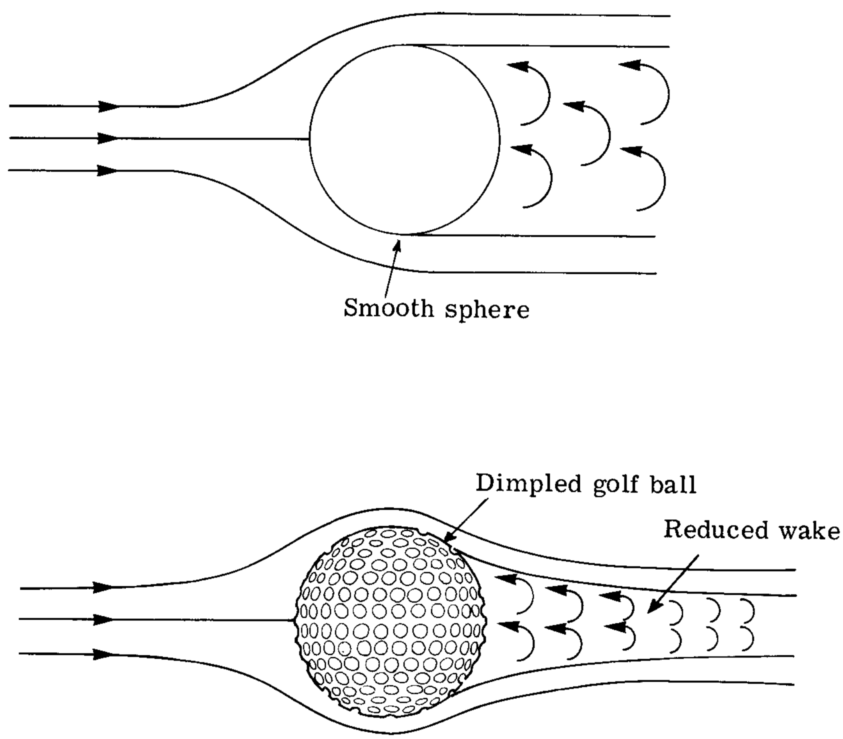

Fig. 4.42 Boundary layer separation

Although the skin friction is increased for a roughened surface, the reduced wake causes a greater drop in forebody drag and a subsequent drop in total drag. At some speeds the drag on the roughened sphere is only about 1/5 of the drag on a smooth sphere. This is the main reason why golf balls are dimpled.

AERODYNAMICS

To achieve a complete understanding of the flight of a projectile it is essential that all the forces, moments and other terms affecting the flight of the missile are represented in well-defined mathematical form.

The Aerodynamic Forces and Moments acting on a Projectile

There are two distinct types of major forces and moments which act on a projectile in flight: they are static forces and moments (drag force, overturning moment) which are dependent on the attitude and translational velocity of the missile, and dynamic forces and moments (Magnus force, damping force, spin damping moment, Magnus moment, damping moment) which are dependent also on the angular velocity. We shall not consider in detail the Magnus force or damping force, but a full treatment is given in (3). Now, when the projectile has zero yaw the only force acting against the projectile motion is the drag force but when the body has an angle of yaw to the relative wind then we have a net cross wind force: this is sometimes called the lift force. Consequently the drag and lift force are in line with and perpendicular to the trajectory respectively. These are known as "wind-axes".

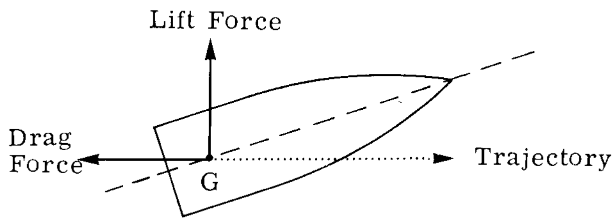

Fig. 4.43 Wind-axes

The American literature commonly refers to the axial force and normal force which are in line with and perpendicular to the body axis respectively. These are called "body-axes".

Since in ballistics we are usually concerned with the motion of the centre of gravity of the projectile along the trajectory we shall use wind-axes. On every axi-symmetric body shape there is a point on the axis where the resultant force appears to act, about which therefore the net moment is zero. This is the centre of pressure, and for incompressible flow it is a function of geometry only; but in compressible flow and especially at transonic speeds the position of the centre of

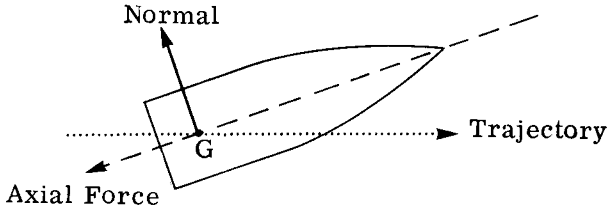

Fig. 4.44 Body axes

pressure is sensitive to Mach number and changes as the Mach number changes. Because the centre of gravity of the body is the point that defines the trajectory it is convenient to make our forces act at the centre of gravity. We can transform the resultant force at the centre of pressure to a couple at the centre of gravity together with a force at the centre of gravity. We then have the drag and cross wind force acting at the centre of gravity. This moment is called the over-turning moment for a shell, which is usually aerodynamically unstable, and the restoring moment for a missile, which is usually aerodynamically stable. We shall call it the yawing moment M and define it as positive with increasing angle of yaw α. It is convenient to use non-dimensional coefficients for describing the forces and moments acting on the projectile. It is usual to non-dimensionalise a force by dividing by $\frac{1}{2}\rho V^2 A_0$, and a moment by $\frac{1}{2}\rho V^2 A_0 . d$ where d is the calibre of the projectile. Thus we have:

$$\text{Drag coefficient } C_D = \text{Drag Force}/\tfrac{1}{2}\rho V^2 A_o$$

$$\text{Cross Wind coefficient } C_L = \text{Cross Wind Force}/\tfrac{1}{2}\rho V^2 A_o$$

$$\text{Yawing Moment coefficient } C_M = \text{Yawing Moment}/\tfrac{1}{2}\rho V^2 A_o . d$$

Variation of aerodynamic coefficients with angle of yaw

C_D Versus α

With an axisymmetric body the variation of C_D is symmetric with α, and has a minimum value for zero yaw. We can write $C_D = C_{D_o} + C_{D_{\alpha^2}} . \alpha^2$ where the term $C_{D_{\alpha^2}} . \alpha^2$ is sometimes called the yaw-drag coefficient and represents the increment in drag coefficient resulting from the angle of yaw.

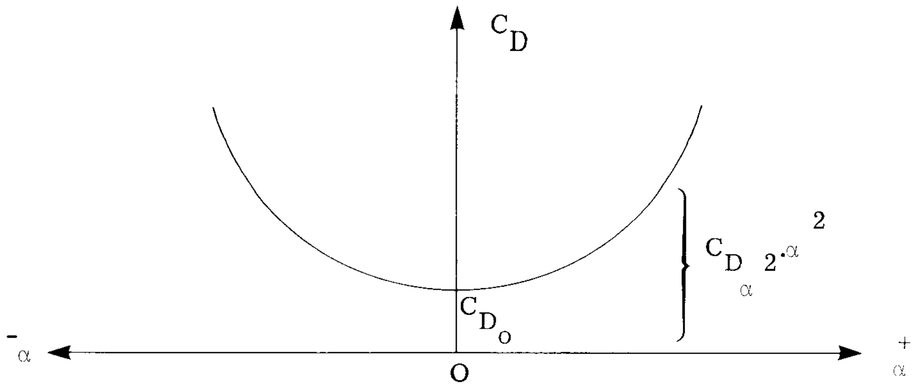

Fig. 4.45 Variation of C_D with angle of yaw

C_M Versus α

For angles of yaw less than about 10^o projectile nose shapes give rise to lift forces proportional to the angle of yaw. Since an axisymmetric body has zero lift for zero angle of yaw the function is a straight line passing through the origin.

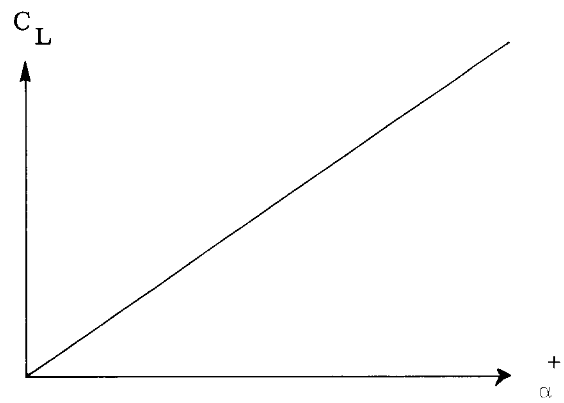

Fig. 4.46 Variation of C_L with angle of yaw

From the equation of a straight line we can write $C_L = (\frac{\partial C_L}{\partial \alpha}) \cdot \alpha$, that is, $C_L = C_{L_\alpha} \cdot \alpha$ where the sub-suffix denotes differentiation with respect to α.

C_L Versus α

Disturbances, such as muzzle motion during projectile exit or cross wind gusts will impart a yawing moment. The yawing moment causes yaw acceleration and the angle of yaw will continue to change even though the disturbance has ceased. If the yaw angle continues to increase, the moment is destabilising and the projectile is aerodynamically unstable. We express this as:

$$\frac{\partial C_M}{\partial \alpha} = C_{M_\alpha} > 0.$$

The centre of pressure of such a body is located between the nose and the centre of gravity. On the other hand, if the derivative $C_{M_\alpha} < 0$, the body is statically stable and the centre of pressure is located behind the centre of gravity. In this case the generated yawing moment strives to decrease the absolute value of the yaw angle and to return the body to its initial position. A damped oscillation commonly occurs. If the derivative $C_{M_\alpha} = 0$, the body is neutrally stable and the centre of pressure coincides with the centre of gravity. This property is often used in wind tunnels and ballistic ranges where the centre of gravity of the model is adjusted until the yawing moment is zero.

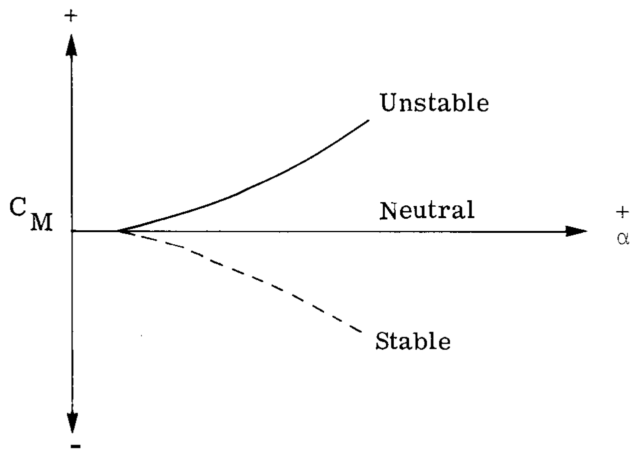

Fig. 4.47 Variation of C_M with angle of yaw

Knowledge of the aerodynamic characteristics of a projectile is essential for the designer. There are many ways of obtaining estimates of the relevant aerodynamic coefficients. The main ones are: 1. theoretical estimation using fluid dynamic theory, 2. empirical estimation using data on similar projectile shapes, 3. wind tunnel testing, 4. range testing, 5. firing trials data, including yaw sonde and radio doppler methods, (see Chapter 7). For most projectile designs almost all of these techniques are used before the design is completed.

Gyroscopic Stability

For reasons given in Section 1 of this chapter, conventional shells are invariably aerodynamically unstable. Nevertheless it is essential that, after leaving the gun, projectiles should continue to point in the general direction of their flight; otherwise drag forces would be unacceptably high and variable side forces would make the trajectory unpredictable and precision would suffer. Flight stabilisation is achieved by spinning the projectile. To see how stability is actually attained it is informative to solve the equations of motion for a spinning top first.

Spinning Top

We consider a symmetrical top, that is, a rigid body which is a solid of revolution. Let one point on the axis of symmetry be fixed at the origin O of coordinates. We then have the situation shown.

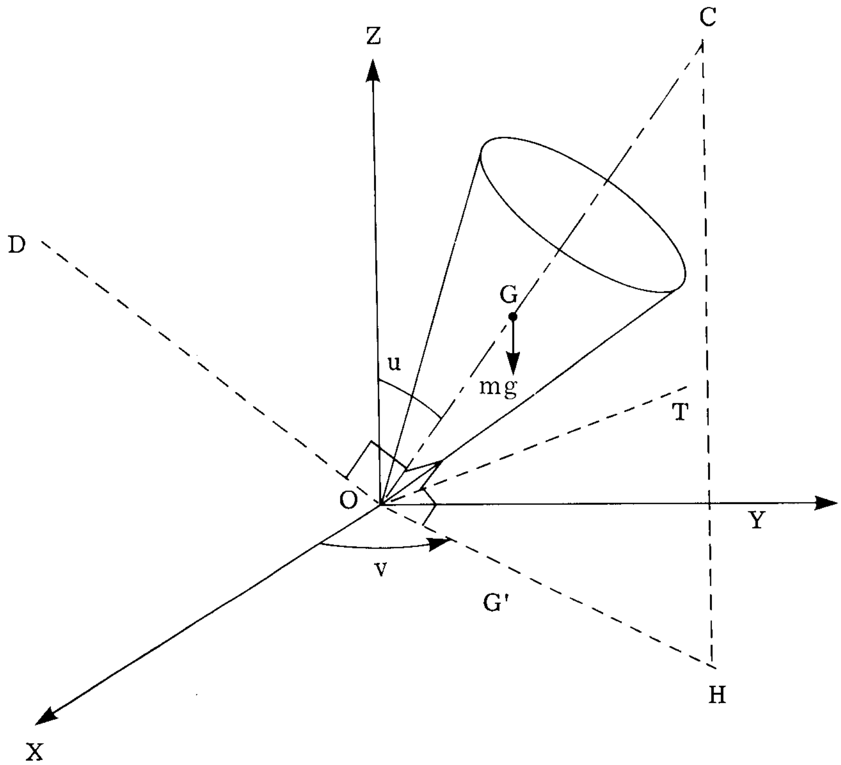

Fig. 4.48 Spinning top

Let mass of top be m.

Let M of I about axis of symmetry (OC) be A(*).

Let M of I about axis perpendicular to OC through O be B.

The forces acting on the top are 1. weight = mg; 2. reactions at the fixed point O. We will ignore any frictional couple at O. Let the top spin with angular velocity ω relative to plane ZOC, i.e. about axis OC. Motion due to change of u is called nutation, motion due to change of v is called precession. The general motion of the top is one of nutation and precession.

(*) M of I; moment of inertia.

Angular Velocities

$\dot{v}$ about OZ, $\dot{u}$ about line perpendicular to OH in plane XOY, i.e. OT and ω about OC (relative to plane ZOC).

Resolve $\dot{v}$ along axes OC, OD where OD is in the plane COH and perpendicular to OC, then

$\dot{v}$ cos u along OC

$\dot{v}$ sin u along OD

(See Fig. 4.49).

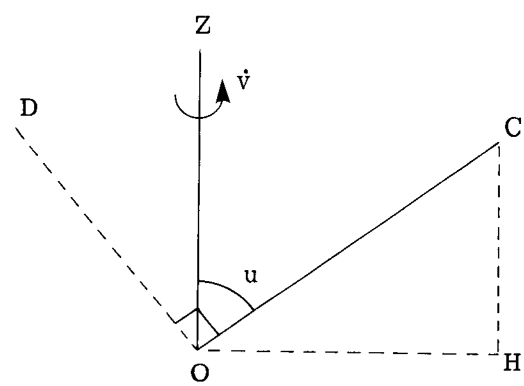

Fig. 4.49 Angular velocities of spinning top

Thus angular velocities are:

1. about OC, $\omega + \dot{v} \cos u$
2. about OT, $\dot{u}$
3. about OD, $\dot{v} \sin u$

The corresponding angular momenta are:

1. about OC $A(\omega + \dot{v} \cos u)$
2. about OT $B\dot{u}$ (1)
3. about OD $B\dot{v} \sin u$

Let

$$N = \omega + \dot{v} \cos u \qquad (2)$$

then N is the rate of spin or total angular velocity about OC.

The components along OC and OD are equivalent to (see Fig. 4.49)

$$\text{about OZ;} \quad AN \cos u + B\dot{v} \sin^2 u$$

$$\text{about OH;} \quad AN \sin u - B\dot{v} \sin u \cos u$$

Finally resolve the components about OH, OT into components about OX, OY to give (see Fig. 4.50)

$$\begin{aligned} \text{about OX:} \quad H_x &= (AN \sin u - B\dot{v} \sin u \cos u) \cos v - B\dot{u} \sin v \\ \text{about OY:} \quad H_y &= (AN \sin u - B\dot{v} \sin u \cos u) \sin v + B\dot{u} \cos v \qquad (3) \\ \text{about OZ:} \quad H_z &= AN \cos u + B\dot{v} \sin^2 u \end{aligned}$$

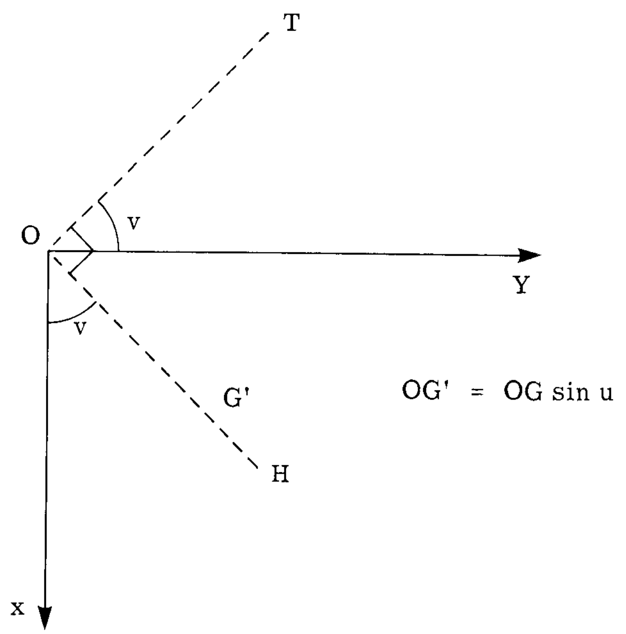

Fig. 4.50 Resolved components of angular velocities

We now have the angular moments measured from fixed axes OX, OY, OZ. Hence we have the equations of motion as (on taking moments about axes OX, OY, OZ)

$$\frac{dH_x}{dt} = - mgl \sin u \sin v$$

$$\frac{dH_y}{dt} = mgl \sin u \cos v \tag{4}$$

$$\frac{dH_z}{dt} = 0$$

where $l = OG$

Equations for small oscillations, i.e. u is assumed small

If u is small then we have $\sin u \quad u$, $\cos u \approx 1$ and so from (3),

$$H_x \cong AN\, u \cos v - B\,(\dot{v}u \cos v + \dot{u} \sin v)$$

$$H_y \cong AN\, u \sin v - B\,(\dot{v}u \sin v - \dot{v} \cos v) \tag{5}$$

$$H_z \cong AN$$

since terms $O(\theta^2)$ are neglected.

Let

$$x = \sin u \cos v \cong u \cos v$$

$$y = \sin u \sin v \cong u \sin v$$

i.e. x, y are the projection of a unit length on OC onto plane XOY.

$$\frac{d}{dt}(ANx - B\dot{y}) = - mgly$$

$$\frac{d}{dt}(ANy + B\dot{x}) = mglx \tag{6}$$

$$\frac{d}{dt}(AN) = O$$

Thus N = constant and, writing $Q = \frac{AN}{B}$, $F = \frac{mgl}{B}$ we have

$$\frac{d}{dt}(Qx - \dot{y}) = - Fy$$

$$\frac{d}{dt}(Qy + \dot{x}) = Fx \tag{7}$$

If we put $z = x + iy$, then

$$\ddot{z} - iQ\dot{z} = Fz \tag{8}$$

If we let $z = We^{\frac{1}{2}iQt}$, then (8) becomes

$$\ddot{W} + (\tfrac{1}{4}Q^2 - F)\,W = 0 \qquad (9)$$

Equation (9) represents simple harmonic motion provided $\frac{1}{4}Q^2 - F > 0$, that is, provided the stability coefficient, s, is such that

$$s = \frac{Q^2}{4F} > 1$$

Now, $s = \dfrac{Q^2}{4F} = \dfrac{A^2N^2}{4B^2F} = \dfrac{A^2N^2}{4Bmgl}$, so that a minimum spin rate $N = \dfrac{\sqrt{4Bmgl}}{A}$ is necessary for stability.

General Stable Motion

Note that we view the top from above. Assume stable motion, that is, $s > 1$. Then (9) has solution

$$W = R\exp\left(i\left(-\sqrt{\tfrac{1}{4}Q^2 - F}\;t + e\right)\right) + S\exp\left(i\left(\sqrt{\tfrac{1}{4}Q^2 - F}\;t + f\right)\right) \qquad (10)$$

where R, S are positive real arbitrary constants

e, f are real arbitrary constants.

We can interpret the motion as follows. The first term in (10) represents an arm OA of length R inclined at an angle f initially and rotating clockwise with rate $\sqrt{\frac{1}{4}Q^2 - F}$. The second term represents an arm S (AB) initially inclined at an angle e to x-axis and rotating with angular speed $\sqrt{\frac{1}{4}Q^2 - F}$ in an anti-clockwise sense. (See Fig. 4.51). The combined motion is that of point B where the arms are joined at A. In general B describes an ellipse in the W-plane with major axis R+S and minor axis $|R-S|$. The period is $2\pi\Big/\sqrt{\frac{1}{4}Q^2 - F}$.

From definition of W and equation (10) we have

$$W = R\exp\left(i\left(\left(\tfrac{1}{2}Q - \sqrt{\tfrac{1}{4}Q^2 - F}\right)t + e\right)\right) + S\exp\left(i\left(\left(\tfrac{1}{2}Q + \sqrt{\tfrac{1}{4}Q^2 - F}\right)t + f\right)\right) \qquad (11)$$

and in the z-plane the angular rate of rotation of the arms are (in an anti-clockwise sense)

$$\text{R arm} : \tfrac{1}{2}Q - \sqrt{\tfrac{1}{4}Q^2 - F}\ \text{slow}$$

$$\text{S arm} : \tfrac{1}{2}Q + \sqrt{\tfrac{1}{4}Q^2 - F}\ \text{fast.}$$

(N.B. we are looking down on top)

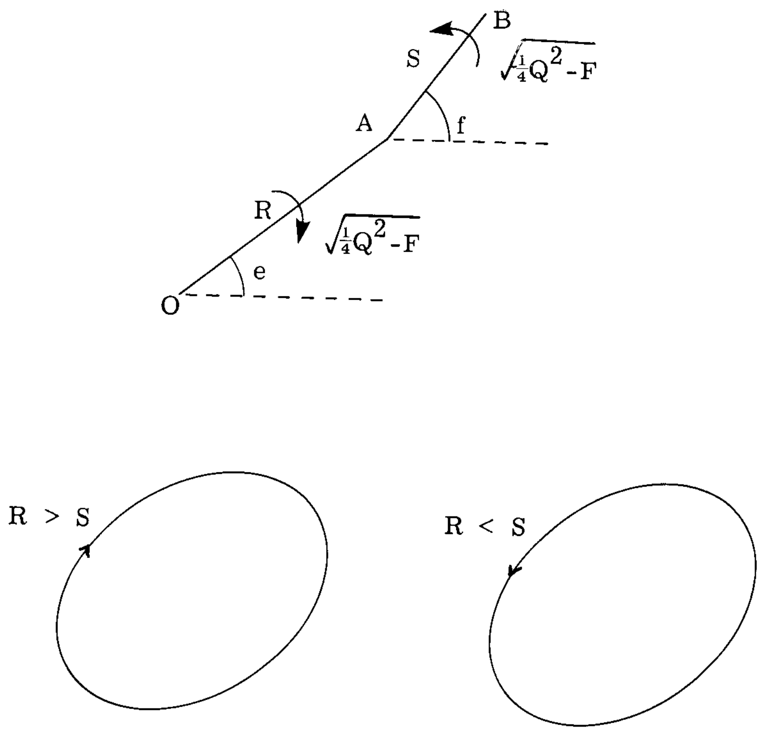

Fig. 4.51 Two arm model applied to a spinning top

THE SPINNING PROJECTILE

Forces Acting on a Shell

In addition to gravity there will be aerodynamic forces acting on the shell, as already discussed. All of the additional forces can be reduced to a single force P acting through the centre of gravity together with a couple M about the centre of gravity. Consider a symmetric shell with axial moment of inertia A and that about any axis perpendicular to axis of symmetry, be B, through the centre of gravity.

We shall assume that the resultant force P lies in the plane of incidence and that the axis of the couple M is normal to this plane. (This is not strictly true but is a reasonable approximation for small yaw). Let α be the angle of incidence, that is, angle between direction of axis of symmetry and direction of motion (GZ). Let GH be perpendicular to GZ and resolve P along GH (lift L) and ZG (drag D).

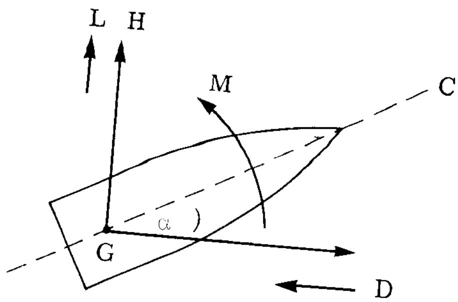

Fig. 4.52 Forces acting on a spinning shell

We assume that the couple M is proportional to α and write (for small α)

$$M = M_\alpha \alpha \quad \text{where } M_\alpha = \frac{\partial M}{\partial \alpha}$$

Note that M_α depends on 1. shape of shell, 2. velocity, 3. change in air density. We shall however ignore 1. and 3. for a given shell and treat M_α as constant. In order to use the previous work on tops we need only note one important result from the theory of dynamics of a rigid body; namely that motion of and motion about the centre of gravity of a rigid body are independent, that is, can be treated separately.

Stability of a Projectile

Let
$$F = \frac{M_\alpha}{B}$$

Then we can use the top equations for motion about the centre of gravity. We note that the couple $M_\alpha \alpha$ takes the place of the weight of the top. Also, certain forces have been neglected, including the Magnus force, Coriolis force (due to the rotation of the earth), the spin reducing couple (due to skin friction), and the damping couple (due to the moment of the air resistance).

The Stability Coefficient of a Shell

The stability coefficient is the same as for the top, that is,

$$s = \frac{A^2N^2}{4B^2F} = \frac{A^2N^2}{4BM_\alpha}$$

and we require $s > 1$. However a more satisfactory value is at least 1.2. Values smaller than this may give rise to undesirably large yaw angles following a disturbance. A value of s around 1.5 is usually favoured. It should be remembered that it is possible to introduce too much stabilisation. Excessive spin rates cause the shell to respond rather slowly to changes in trajectory direction and the axis of the shell tends to remain in its initial direction. To find N for a spinning shell we have, if the rifling is 1 in n (that is shell turns one complete revolution in a distance of n calibres), that the shell turns 2π radians in a distance 2nr, where r is the radius of the shell, and if it emerges at a speed V the rate of turn is

$$N = \frac{2\pi V}{2nr} = \frac{\pi V}{nr} \quad \text{rad/sec}$$

and so,

$$s = \frac{A^2 N^2}{4B^2 F}$$

$$= \frac{A^2 \pi^2 V^2}{4n^2 r^2 B^2 F}$$

$$= \frac{A^2 \pi^2 V^2}{4n^2 r^2 B M_\alpha}$$

To express the stability coefficient in terms of the more fundamental properties of the shell and gun, we may express the yawing moment derivative M_α in terms of the yawing moment coefficient derivative C_{M_α}: thus

$$M_\alpha = \tfrac{1}{2}\rho V^2 A_o d . C_{M_\alpha}$$

where d is the calibre. Then

$$s = \frac{8\pi}{\rho n^2 d^5} \frac{A^2}{B} \cdot \frac{1}{C_{M_\alpha}},$$

where we have written $A_o = \frac{1}{4}\pi d^2$. It is apparent that the worst condition for the gyroscopic stability is when ρ and C_{M_α} are at a maximum. We know that atmospheric density ρ is at a maximum at ground level, so the value of s will be lowest at launch for the charge corresponding to the Mach number for which C_{M_α} is maximum. For a typical shell the C_{M_α} against Mach number curve is of the following general form.

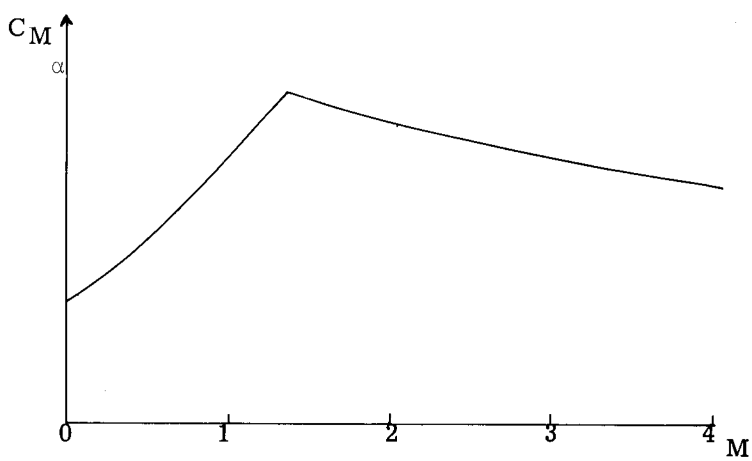

Fig. 4. 53 Variation of C_{M_α} with M

Thus the gyroscopic stability is least during a transonic launch, that is, at a muzzle velocity at or near the speed of sound, approximately 340 m/s. As the diameter and mass of the projectile are usually fixed, as far as the aerodynamics is concerned, A, the axial inertia is also nearly fixed, hence gyroscopic stability must be attained mainly by varying N/V, that is, $2\pi/nd$, B and C_{M_α}. Overall, these reduce to varying the length to diameter ratio of the shell. This is the reason why there is a maximum length of shell, in terms of calibres, that can be gyroscopically stabilised with a given rifling twist.

Angular Motion of Spinning Projectiles (Two Arm Model)

If we refer to our previous analysis of the spinning top we saw that by looking down on the top the motion could be interpreted as two arms; one slow and the other fast. This same procedure can be applied to our spinning projectile and now the locus of the nose of the spinning projectile can be interpreted in terms of two rotating arms, OA, AB, hinged at A, O being fixed. The slow arm rotates with angular velocity

$$\frac{AN}{2B} - \sqrt{\left[\left(\frac{AN}{2B}\right)^2 - \frac{M_\alpha}{B}\right]}$$

and the fast arm AB rotates with angular velocity

$$\frac{AN}{B} + \sqrt{\left[\left(\frac{AN}{2B}\right)^2 - \frac{M_\alpha}{B}\right]}$$

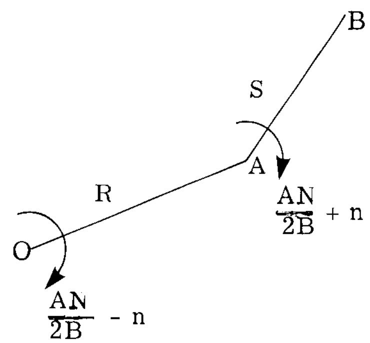

Fig. 4.54 Two arm model applied to spinning projectile

The rates of rotation may be written more concisely as AN/2B-n, AN/2B+n respectively. The directions shown on the diagram would represent the angular motion viewed from the rear, the spin being in the clockwise direction (from the rear).

Some Typical Trochoidal Motions

The application of the two arm model provides the means to generate typical angular motions. The general shape of the locus of the nose is sometimes termed a trochoid. The particular pattern is determined by 1. the ratio of AN/2B to n, that is, the balance between spin rate and aerodynamic moment; 2. the ratio of the two arms, which depends only on the initial disturbance. For convenience, we shall denote the 'lengths' of the fast and slow arms by R, S, respectively. Typical patterns for an aerodynamically stable projectile are shown overleaf.

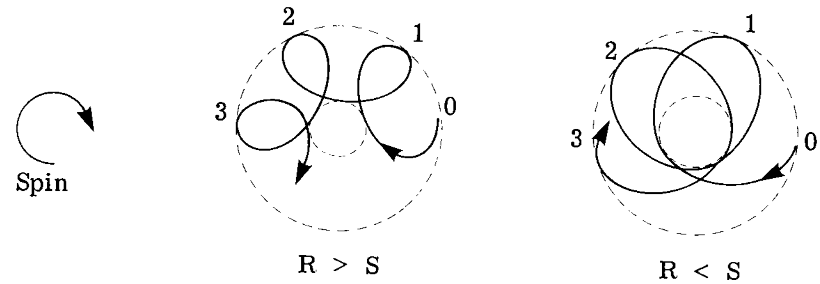

Fig. 4.55 Typical patterns for aerodynamically stable projectiles

The number of loops in the complete circle depends on the ratio of AN/2B to n, and increases as their ratio approaches one, that is, as the aerodynamic influence decreases. Typical patterns for an aerodynamically unstable projectile are:

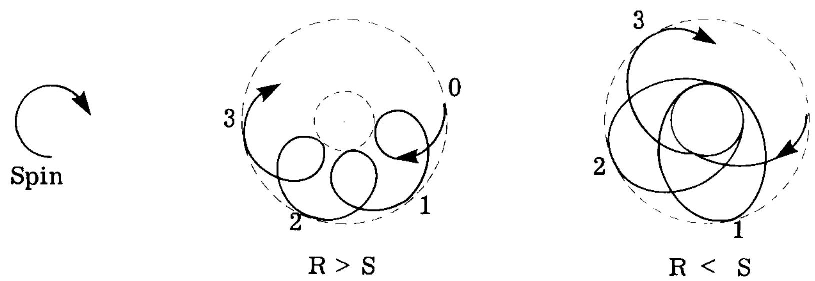

Fig. 4.56 Typical patterns for aerodynamically unstable projectiles

Again, the number of loops increases as the ratio of AN/2B to n approaches one, that is, as the aerodynamic influence decreases. It will be noticed that the looping is outward for the stable and inward for the unstable projectile, but in both cases the motion is confined between two circular boundaries.

Rosette Motion

The type of motion most typical of a projectile from a gun is given by $R = S$ and corresponds to the ellipse degenerating into a straight line. Such a motion is sometimes called "rosette" motion. The angle of yaw is initially zero and repeatedly passes through the zero value. Examples are given at Fig. 4.57.

Steady Precessions

Appropriate initial conditions could result in a steady precessional motion, the locus in this case being a circle. Two precessional motions are possible. If the length of the slow arm is zero ($R = 0$) we get the "fast precession" at rate

$$AN/2B + n$$

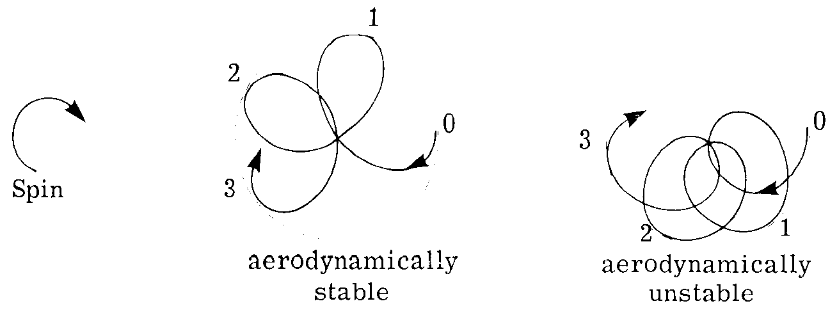

Fig. 4.57 Rosette motion

in a clockwise direction (for clockwise spin). If the length of the fast arm is zero (S = 0), we get the "slow precession", at rate

$$AN/2B - n$$

The direction will be anti-clockwise for the aerodynamically stable projectile and clockwise for the unstable one.

Equilibrium Yaw Motion

As discussed in Part 1 of this chapter, the aerodynamically unstable projectile will fly with an average angle of yaw to the right of the trajectory. The trochoidal motion described above is retained, but its centre is displaced by the amount of the equilibrium yaw. For an aerodynamically stable missile, the equilibrium yaw will be to the left of the trajectory since the sign of M_α is reversed. Typical motions for the two cases are illustrated below.

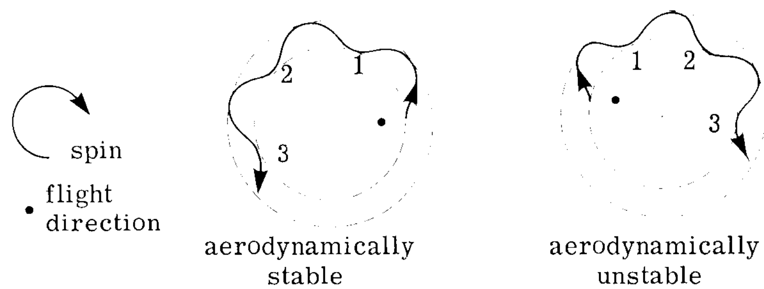

Fig. 4.58 Equilibrium yaw motion

Effect of Aerodynamic Damping

In representing the angular motion in terms of a looping movement between two fixed circular boundaries, the effect of aerodynamic damping has been ignored. If damping terms are included in the equations, the consequences are as follows:

1. the spin rate declines very slowly, 2. the fast arm is strongly damped, 3. the slow arm is weakly damped for the aerodynamically stable projectile and weakly negatively damped, that is, slowly increases, for the unstable projectile.

Equilibrium Yaw and Drift

If the trajectory were a straight line, the spin stabilisation would prevent the yaw angle growing following any disturbances, and damping moments would in time reduce the amplitude of the motion. Near the muzzle of the gun the curvature of the trajectory is small but, as the shell proceeds, the curvature increases, reaching a peak at the vertex. The spin stabilisation then has the effect of ensuring that the average direction of the shell axis turns with the trajectory: the axis will of course oscillate about this mean direction. For the average direction of the shell axis to follow the curve of the trajectory, the angular momentum vector I must slowly rotate. This implies that increments ω of angular momentum must be continually added to I in the downward direction.

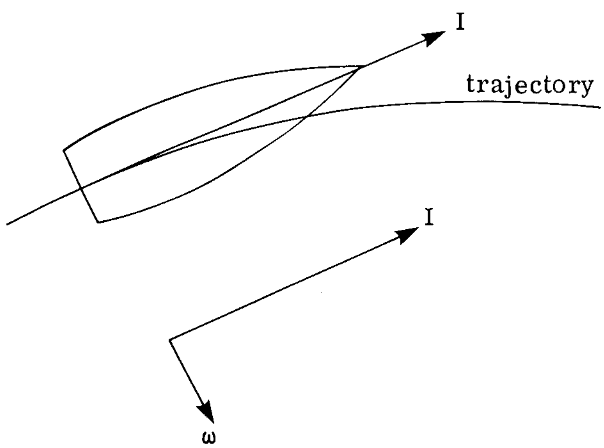

Fig. 4.59 Increments of angular momentum

Viewed from the top, the continuous addition of angular momentum ω requires a continuous yawing moment M in the direction shown below. This can only be produced if the shell flies on the average with a small angle of yaw to the right of the trajectory. This is called equilibrium yaw.

The increment ω in time dt may be written as $\omega = I\,d\psi$, where ψ is the inclination of the trajectory to the horizontal. It follows that

$$M = \frac{d\omega}{dt} = I\frac{d\psi}{dt}$$

Then writing

$$M = M_\alpha \alpha,$$

$$I = AN,$$

the equilibrium yaw α is given by

$$\alpha = \frac{AN\dot{\psi}}{M_{\alpha}}$$

In terms of the stability coefficient s of the shell, the equilibrium yaw may be expressed as

$$\alpha = \frac{4sBg \cos \psi}{ANV}$$

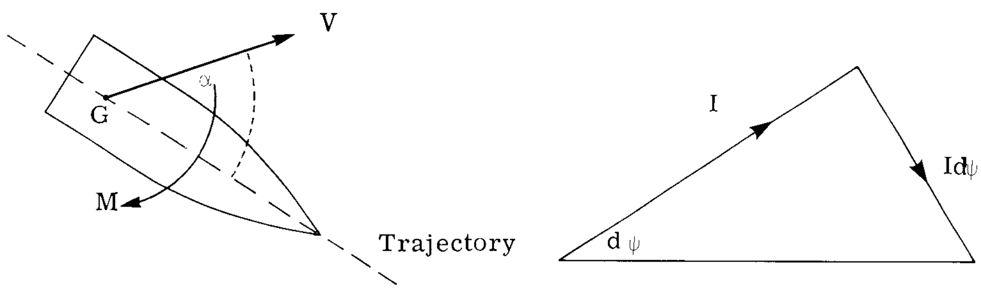

Fig. 4.60 Equilibrium yaw angle

The derivation of this formula can be found in (8). We have seen that by spinning a shell at the appropriate rate, gyroscopic effects can be used to keep the nose pointing approximately along the trajectory. However, the shell inevitably suffers some yaw and the presence of equilibrium yaw will mean that the nose is pointing to the right of the trajectory. The air meeting it during flight is thus deflected to the left, so creating a cross wind force to the right which will mean that the shell will drift to the right in flight. The drift increases with the time of flight and, for a given muzzle velocity, the amount is found to vary with the angle of projection θ, according to the law:

$$D = C \tan \theta$$

where the constant C can be found experimentally.

NEXT PHASE

We have now looked at the ballistics involved in launching a projectile and in its flight to the target. The next stage is concerned with its performance when it reaches the target. This is called terminal ballistics and is the subject of the next chapter.

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SELF TEST QUESTIONS

QUESTION 1 Assuming in-vacuo motion, what would be the maximum range of a projectile fired with a muzzle velocity of 220 m/s?

Answer ....

QUESTION 2 What principle enables the use of wind tunnels to predict the aerodynamic forces acting on an actual projectile in flight?

Answer ....

QUESTION 3 For two geometrically similar shapes, what does equality of Reynold's number imply?

Answer ....

QUESTION 4 Why are golf balls dimpled?

Answer ....

....

QUESTION 5 What is the centre of pressure?

Answer ....

....

....

....

QUESTION 6 When does the centre of pressure coincide with the centre of gravity?

Answer ....

....

....

QUESTION 7 If a 5.56 mm projectile, with a muzzle velocity of 990 m/s travels a distance of 30.48 cm in one complete revolution, estimate the spin rate of the projectile.

Answer ....

....

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. . . .

. . . .

QUESTION 8 How does the drift on a projectile vary with the angle of projection?

Answer . . . .

. . . .

ANSWERS ON PAGE 204

5.

Terminal Ballistics — Part I

SCOPE

Terminal ballistics may be defined as the study of the effects of projectiles on a target. The conditions under which missiles impact against targets vary widely, depending on strike velocity, strike angle, and the type of projectile and target. In this chapter we shall be mainly concerned with the terminal ballistics of projectiles against armour.

VELOCITY RANGES

The range of initial velocities is, in many ways, the most fundamental consideration because velocity affects the variety of impact phenomena so much that it can override almost any other consideration. The velocity range for a projectile fired from a conventional gun is 500 to 1300 m/s, the nominal ordnance range. The ultraordnance domain from 1300 to 3000 m/s is represented by warhead fragments: and above this upper limit is the hypervelocity range, involving shaped charges.

ANGLE OF ATTACK

An important consideration in terminal ballistics is the angle at which the projectile strikes the target. In the United Kingdom the "angle of attack" is the angle between the path or line of arrival of the projectile and the "normal" angle to the plate under attack: it is shown as angle θ in Fig. 5.1. The normal angle is defined as being 90° to the target plate. The angle of attack as defined here is not common to all countries and some use the angle α as the angle of attack. The actual angle of attack will be dependent upon a number of factors such as the stability of the projectile in flight and the type of projectile and target.

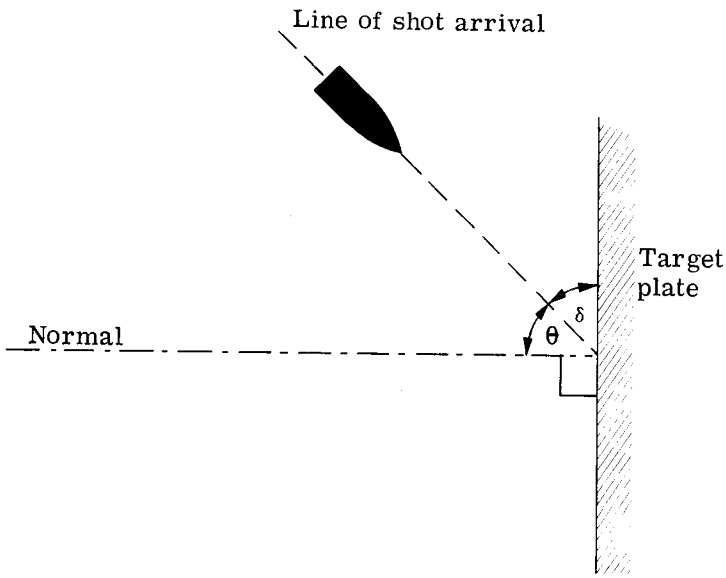


Fig. 5.1 Angle of attack

CHARACTERISTICS OF PROJECTILES

The penetrator shape is significant in determining the mode of target failure. Pointed penetrators exhibit a piercing action in which target failure centres about the projectile axis. Blunt shapes, on the other hand, exhibit a plugging mode of perforation (see Fig. 5.3). The transition depends on penetrator shape, and it is important to note that the effect of sharp and blunt shapes plays an important role in establishing penetration resistance. The criterion for "sharp" and "blunt" shapes is often determined by the ratio of nose length/calibre. If this ratio is greater than or equal to unity then it is termed sharp; if the ratio is less than unity, then it is termed blunt.

To maximise penetration it is generally desirable for a penetrator to be long and dense. There are basically two disadvantages that appear with increased length: the first is an increased chance of bending mode failures, and the second is an external ballistic instability for spin-stabilised projectiles. However, projectile designs that use fin stabilisation avoid the latter problem.

Deformation of the projectile increases the diameter of the projectile and therefore decreases the potential for penetration. Consequently, the penetrator should ideally be difficult to deform. Steels, forms of tungsten carbide and depleted uranium are all suitable for use as penetrators. The latter substances have the advantage of high density, but brittleness defines definite limits on the range of impact conditions for which these heavy materials can be used. Steel is reasonably dense and can be given considerable toughness; its comparative cheapness and availability are distinct advantages.

CHARACTERISTICS OF TARGETS

Roads, bridges, factories, tanks, ships and helicopters are all types of targets. However, during an impact it is usually only an element of the target that is attacked; for example an armour plated tank wall. Therefore, one convenient way of classifying targets is to relate to their relative thicknesses. A "thin target" assumes that for all practical purposes, stress and deformation gradients do not exist. If the rear surface of a target exerts considerable influence on the deformation process during all or nearly all of the penetration then it is termed an "intermediate target". A "thick target" is defined as one in which the rear boundary influences the penetration process only after substantial travel into the target element, and lastly, a "semi-infinite" target is one in which there is no influence of a rear boundary on the penetration process. Another method of classification of targets is their geometry. The target, if thin, may consist of single plates, sandwich plates, spaced plates; the shape may be flat, curved, or irregular.

An important part of penetration mechanics is an assessment of the material characteristics of the target. Penetrability of substances is a quantitative measure of particular interest to ballisticians. A comparison of performance of a single projectile type against a spectrum of materials will establish some type of order among them. Comparative penetrability is the basis for a crude classification of target materials as low resistant which mainly consist of soils, moderately resistant such as concrete and low-strength metal alloys, and highly resistant, including the high-strength metals, alloys and ceramics. These classes tend to correspond to different penetration phenomena, require different investigative approaches and correspond very roughly to the present classification of targets by relative thickness and density.

The behaviour of a specified material is usually represented in terms of a model that depends on its state and character. Most targets are characterised as solids, but in some instances the target may act like a fluid. This happens when solids are affected by the extreme pressures produced by hypervelocity impact. Solid models include one or more of the domains of elastic, plastic, viscous, or hydrodynamic behaviour. Furthermore, an accurate specification of material response of any solid is complicated by the extreme range of stresses to which an object is subjected during penetration and by the diversity of deformation and failure patterns that occur.

Due to the different types of projectile and target characteristics, it is easily seen that forces of many different kinds can be induced in missiles when they strike their targets. Projectiles will react to these induced forces in a variety of ways. On impact, they may perforate the target, may penetrate the target, and they may ricochet.

ATTACK OF ARMOUR

Penetration and Perforation

The defeat of a target often involves penetration or perforation of it by a missile,

so it is useful to distinguish between penetration and perforation. Penetration may be defined as the entrance of a missile into a target without completing its passage through it; perforation usually implies the complete piercing of the target by the projectile.

Penetration

In general, the high-speed interaction of a long rod penetrator with a target can be divided into three phases. The first phase is the impact phase where stress waves and stress levels are critical to initiate penetration. The second phase is penetration where either or both the target and penetrator behave as though they were fluid. This hydrodynamic failure is produced in those regions where the pressures involved are well above the yield strength of the materials. The third phase is the perforation phase which is discussed under its own heading later in the chapter.

The interaction between a target and a penetrator is an interaction between materials and can be explained in terms of the science of materials. The impact phase is complex and often ideal situations are assumed. Idealised theories may provide useful conclusions, and modifications may be subsequently made to improve the model. The model may consider wave reflections in the penetrator and stress waves in the target, for example. Wave motion within the penetrator and target can be accepted for low speeds, where there is no yield of penetrator material to be considered. As the impact velocity increases, the penetrator material begins to yield, the material flows plastically and penetration starts. The impact phase may have important effects on the penetrator. Shortly after impact, the projectile suffers a radial tensile stress and the striking end or nose of the rod mushrooms radially while pushing into the target. At the same time, a plastic wave travels towards the free end of the projectile. In some cases, when the impact stress is high enough, the rod or body of the penetrator will fracture as the applied stress reaches the ultimate tensile stress of the material. Such a stress level initiates axial cracks at the periphery of the mushroomed region at the nose of the rod. The cracks propagate and shatter at the stem of the rod or body. So, as the plastic wave travels down the rod it leaves shattered material in its path.

In summary: at the moment of impact a stress wave is initiated in both the target and the rod or body of the projectile and a very high level of stress is applied on the impact area. The high stress level produced initially will produce a crater and the projectile will penetrate the face of the target.

After the first phase of impact when the stress has been weakened and has progressed away from the crater, an almost steady state is set up in which the crater is being deepened at constant speed. During this phase the hydrodynamic analogy proves useful because the pressures at the bottom of the crater are well beyond the yield strength of the material in both the penetrator and the target when the velocity is high. During penetration, the projectile may be partly shattered; it will also have become plastic. Its nose will be advancing with a specific velocity V and the crater may be imagined as advancing in an opposite direction at a velocity U where V is greater than U (see Fig. 5.2). The crater progresses by ductile failure and the target material flows aside. The pressure created and the energy transferred by the impact melts the front of the projectile and the bottom

of the crater. The penetrator and target material behave as fluids although not as perfect fluids. Because the back of the rod travels faster than the front of the rod which in its turn travels at the speed of advance of the crater, it follows that the projectile is constantly eroded. The material being eroded is forced backwards, relative to the bottom of the crater, and flows between the crater wall and the remaining portion of the penetrator. The projectile is thus being consumed and turned into a hollow cylinder as illustrated. In summary, provided the speed of impact is high enough, the projectile flows plastically exerting a pressure on the bottom of the crater thus deepening the crater. This action causes the projectile to be inverted and turned into a hollow cylinder.

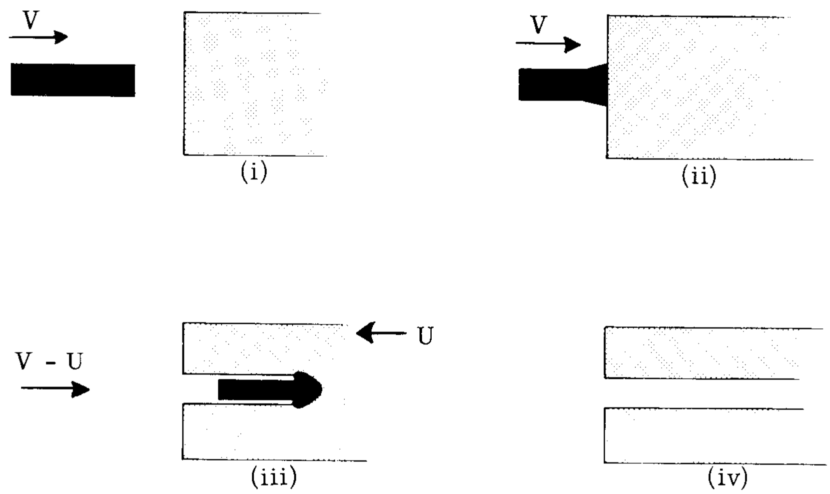
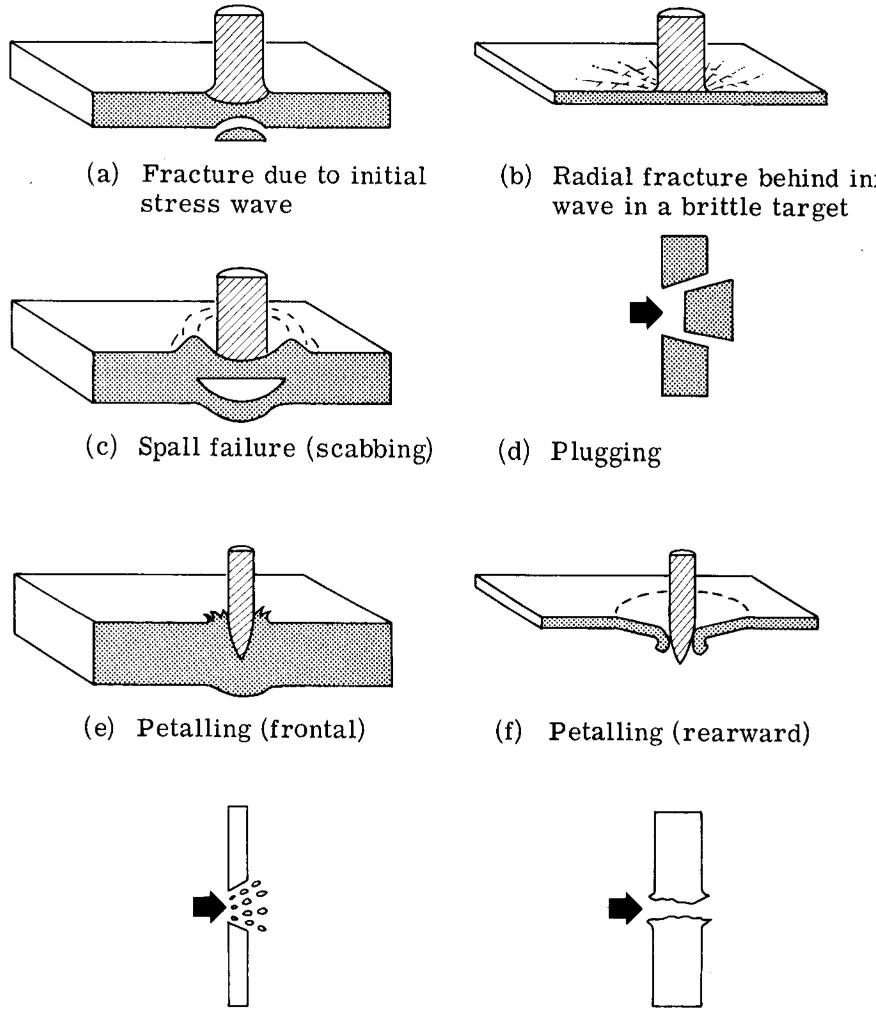


Fig. 5.2 Impact of a long rod penetrator

Perforation of Armour

The process of perforation is a complicated mechanism, which has not yet been fully explained in the theoretical sense. In this section we will look at some of the observed features encountered in these processes.

Plate failure is due to the interaction of a variety of mechanisms with one predominating, depending on material properties, geometric characteristics and impact velocity. The most frequent types, shown in Fig. 5.3, consist of fracture, radial fracture, spalling, scabbing, plugging, front or rear petalling, or fragmentation and ductile hole enlargement.



(a) Fracture due to initial stress wave

(b) Radial fracture behind initial wave in a brittle target

(c) Spall failure (scabbing)

(d) Plugging

(e) Petalling (frontal)

(f) Petalling (rearward)

(g) Fragmentation

(h) Ductile hole enlargement

Fig. 5.3 Perforation mechanisms

Fracture

Failure involving fracture results in the perforation of thin or intermediate targets. Fracture due to initial stress waves, which are stronger than the ultimate compressive strength of the target, could typically occur in weak, low density materials. Radial fracture would be limited to brittle targets such as ceramics.

Spall Failure (Scabbing)

Scabbing is a material failure due to the reflection of the initial compressive wave from the far side of the plate and is a commonplace phenomena under explosive loading, a good example of this is the action of High Explosive Squash Head (HESH).

Plugging

Plugging develops when a nearly cylindrical slug of approximately the same diameter as the penetrator is set in motion by the projectile. Failure occurs due to shearing produced around the moving slug. Plugs are most likely to be found in very hard plates of moderate thickness. Its presence most frequently occurs when blunt penetrators are used and it is sensitive to velocity and angle of attack.

Petalling

Petalling is most frequently observed in thin plates struck by ogival or conical penetrators at relatively low impact velocities or by blunt projectiles near the ballistic limit. As the material in the bulge on the back of the plate is further deformed by the projectile, the elastic properties of the armour are eventually exceeded and a star-shaped crack develops around the tip of the penetrator. The sectors subsequently formed are then pushed back by the motion of the projectile, forming petals.

Fragmentation

Fragmentation occurs when the target is composed of brittle material. The fragments generated by a failed target themselves act as projectiles and must be considered as penetrators when meeting any subsequent target.

Ductile Failure

The ductile type of failure is the kind most commonly observed in thick plates. The perforation is accomplished by radial expansion of the plate material as the projectile pushes through.

Predicting Failure

Numerous theoretical models have been developed in attempts to predict the perforating ability of a projectile without conducting experimental test firings. The behaviour of armour is so complex that none of these models is completely satisfactory. In most of the theoretical developments, the volume of the hole produced by the impacting projectile is assumed to be proportional to the kinetic energy lost by the projectile when it perforates the plate.

TYPES OF PROJECTILES

The first and still very common type of armour defeating projectile is some form of solid shot which depends upon kinetic energy to penetrate armour. More recently two forms of chemical or, more descriptively, explosive attack, have been developed. The first is called High Explosive Anti-Tank (HEAT), which employs a shaped or hollow charge principle to punch holes through armour. The second is High Explosive Squash Head (HESH) which is a soft-nosed high explosive shell: it pancakes on the armour before detonating and causes a scab to blow off the inside.

Kinetic Energy

The kinetic energy form of attack is to use a solid projectile or 'shot' to impart as much energy as possible on the target concentrated over as small an area as possible. That is, it is desirable to maximise the relationship MV^2/d^2, or in other words the projectile should be a long thin rod. However, there is a conflict between this requirement for shot shape, size and mass at the target and the shot requirements in the gun and in flight. In the gun the shot should present the largest possible cross-sectional area against which the propellant gases can act, and it should be light. Therefore the shot should have a high value of d^2/M and so should be a short squat projectile made of a low density material. In flight, to ensure the maximum kinetic energy is delivered at the target, it is important that the shot loses as little velocity as possible on its way to the target. A heavy dense projectile has a better carrying power than a lighter one, and hence will have the potential for greater ranges. Furthermore, a thin projectile with a small cross-sectional area will maintain its velocity better than one with a large cross-sectional area. So a long thin dense shot is the requirement for the shot in flight. Also, the longer and thinner the projectile provided that it is kept as heavy and dense as possible, the better its penetrative performance. The depth of penetration for a projectile that survives impact without fracture is approximately proportional to the impact velocity and the projectile length, and it increases with the density of the penetrator. However, very long thin dense projectiles striking armoured plate at high speeds are likely to ricochet and break-up due to material failure. (1) gives a good practical account of the various forms of shot failure. The differing requirements have been reconciled in the design of an Armour Piercing Discarding Sabot (APDS) shot.

Until the adoption of APDS, various armour piercing projectiles were commonly used and they were generally modified in some way to overcome the conflicting

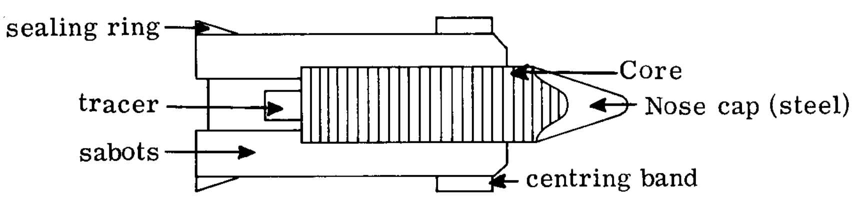


Fig. 5.4 Basic components of an APDS projectile

requirements outlined above. The current APDS consists of a small, high density tungsten carbide core, sheathed in a sabot of light alloy which serves to convey the sub-calibre shot through the gun bore and out of the muzzle. Having performed this function, the sabot conveniently breaks away, reducing air resistance and leaving the high velocity shot to expend its energy in penetration of the target where its smaller calibre results in less dissipation of energy. Because of their considerable velocities, with muzzle velocities in the range of 1378 m/s, such projectiles have a high degree of accuracy and can usually be employed at long ranges. To provide stability, the APDS round is spun and must be fired from a rifled gun. For a kinetic energy round there are limitations in this method, because the core cannot be made too long before it becomes unstable: penetration of armour could be improved by making the core longer and thinner and this can be achieved by fin stabilisation. The APFSDS can also acquire higher muzzle velocities due to the use of a smooth bore gun.

An illustration of the major components of an Armour Piercing Fin Stabilised Discarding Sabot (APFSDS) round is shown in Fig. 5.5. However, as all the energy the kinetic energy shot possesses at the target has to be developed and imparted in the gun, there are a number of penalties to be paid for this. For example, a large and heavy gun is needed to absorb the recoil energy, and there is an increased effect of barrel wear compared with guns with lower velocity projectiles.

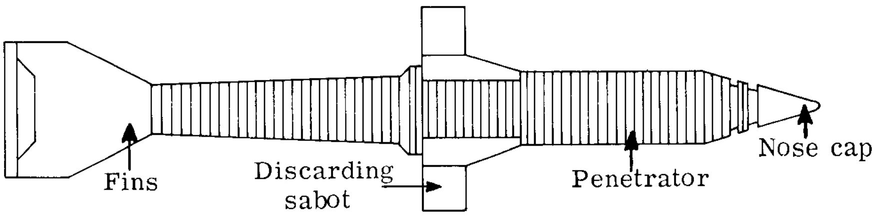


Fig. 5.5 Basic components of an APFSDS projectile

Chemical Energy - Explosive Projectiles

Chemical energy in the form of explosive projectiles can be used to attack armour in a number of ways; not all are equally effective. The targets attacked by high explosive (HE) shells may be primarily damaged by fragments, by blast, or by both. The usual type of HE fragmentation shell is illustrated below in Fig. 5.6.

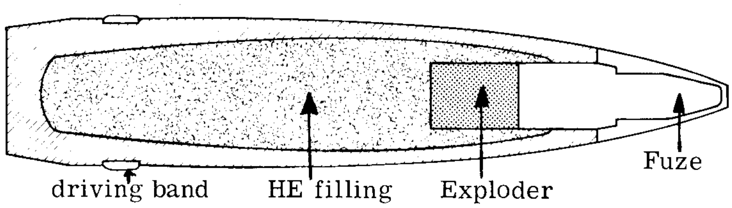


Fig. 5.6 Basic components of a HE fragmentation shell

The shell consists of a hollow streamlined projectile filled with high explosive and provided with a fuze and exploder system so that it may be detonated at the right time and place in relation to the target. As the detonation proceeds through the explosive, the metal case begins to swell and to break up into a large number of pieces of varying sizes which are thrown outward at high velocity. These fragments are effective against personnel and against such materials as guns, soft-skinned vehicles, and aircraft, provided a vital part is hit.

The direction which the fragments take will be governed largely by the shape of the projectile. For a spherical projectile when detonation starts at the centre, the distribution of fragments should be uniform. For cylindrical projectiles most of the fragments would be concentrated in a fairly narrow beam of side spray. The maximum fragmentation effect is created when the greatest number of fragments of the maximum mass and velocity are produced. If the number of fragments is increased the chances of a hit on a target at a given distance from the burst will increase. But, since the average mass of a fragment will be correspondingly reduced, the initial kinetic energy will be less and their velocity will fall of more rapidly. In general a fragment from a modern shell will travel at about 1000 m/s and will weigh between 1.2g and 3.75g. The distribution in mass of the fragments will depend upon many factors such as wall thickness, the ratio of the weight of explosive to the weight of metal, and the type of metal. The velocities of the fragments will be determined by the same sort of factors, but in general a thin-cased shell will give rise to fragments having high velocities. Fragmentation bombs and shells, mortar shells, grenades, mines, rockets and other fragmentation missiles are all roughly similar in nature. They differ primarily in the specific shapes used, the thickness of the shell wall, and in the amount of explosive used.

Blast is the shock caused by the detonation of the high explosive filling. The effects are most pronounced if the shell penetrates the surface of a target before detonation. In the open environment, blast is effective for relatively small distances and fragmentation has more effect. The main practical uses of blast are in the destruction of buildings and structural damage in general. Conversely the use of straightforward high explosive shells with their blast and fragmentation effects is not really effective against tanks. HESH and HEAT are, and we will now discuss these in some detail.

Shaped Charges

The energy of an explosion can be concentrated by properly shaping the charge. Early in World War II, it was discovered that when a hole in the explosive was lined with a thin metal layer, the damage produced by the charge was greatly increased. Hollowed lined charges are variously referred to as hollow charges, shaped charges, lined charges and Munroe charges. Whatever its nomenclature, it consists basically of a hollow liner of inert material; this is usually metal of conical shape. The general effect of lining the charge is shown in Fig. 5.7.

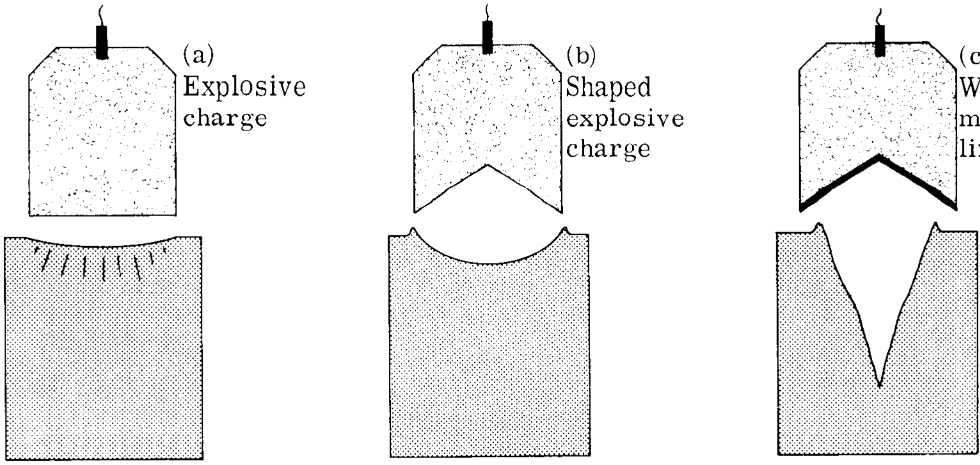


Fig. 5.7 The shaped charge principle

Figure 5.7 (a) shows the effect produced by an ordinary cylindrical charge with no cavity; Fig. 5.7 (b) shows the effect produced by a shaped but unlined charge, and the third, Fig. 5.7 (c), the effect produced by a shaped and lined charge. This effect is due to the fact that when a slab of high explosive material is detonated, the detonating wave leaves the surface of the material at an angle. Therefore, if the high explosive is formed with a cone-shaped hollow, the detonating impulses emanating from the inner surfaces of the core will be focused and concentrated at an external point on a line along the axis of the shell. The liner collapses under the action of the explosive and gives rise to a stream of high velocity, high density material that is capable of considerable penetration. The conical liner is usually broken up by the explosion into three components; a plug, a jet and fragments.

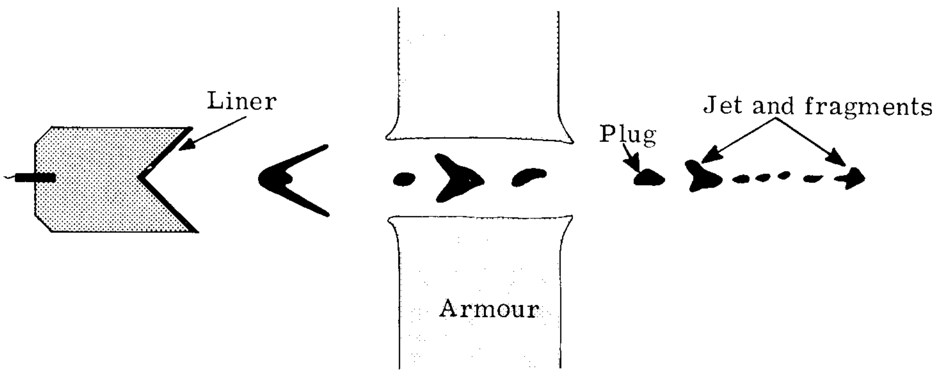


Fig. 5.8 Penetrative performance of a conical liner

Figure 5.8 is a diagram of the mode of deformation and break up of such a conical liner.

High Explosive Anti-Tank (HEAT) Effect

In principle, HEAT works by using the shaped charge effect; that is by using the energy available from the detonation of a charge of high explosive to collapse and break up a metal liner into a metallic jet and plug.

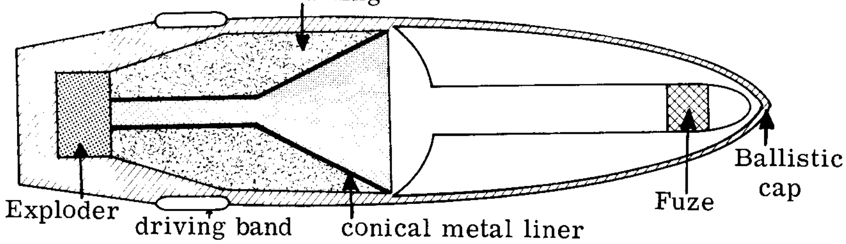


Fig. 5.9 Basic components of a HEAT projectile

When making a HEAT projectile the HE filling is formed with the conical recess in the forward end, around a liner, which is normally made of metal. The nose of the shell is hollow and strengthened so as to ensure a stand-off distance when the filling detonates, which will bring the point of maximum concentration of force against the surface of the target. The shell fuze acts immediately it is decelerated and on detonation the tremendous release of energy creates a pressure wave.

The wave crushes the cone in upon itself. The cone is further deformed so that a high velocity jet of metal and gas is squeezed out along the axis of the cone by the pressure wave. The stand-off distance is determined to ensure that the pressure wave jet of gas and metal is fully formed at the point of impact with the armour target. Penetration of the armour is dependent on many factors, the major ones are stand-off distance, the type of explosive and cone angle, and the diameter and thickness.

About 20% of the metal liner goes into the metallic jet which has a velocity gradient from its tip of approximately 8000 to 9000 metres per second, to its tail of about 1000 metres per second. The remaining 80% of the liner forms a plug which follows the jet at a much lower velocity of the order of 300 metres per second. The jet achieves its penetration solely by the intense concentration of kinetic energy at its tip which exerts a pressure of about 200 tons per square inch (3050 mega pascals) on the armoured plate. Under this intense strain the plate gives radially and in doing so is permanently deformed. The penetration achieved by a HEAT projectile is relatively good, and a small amount of high explosive charge can penetrate a considerable thickness of armour plate.

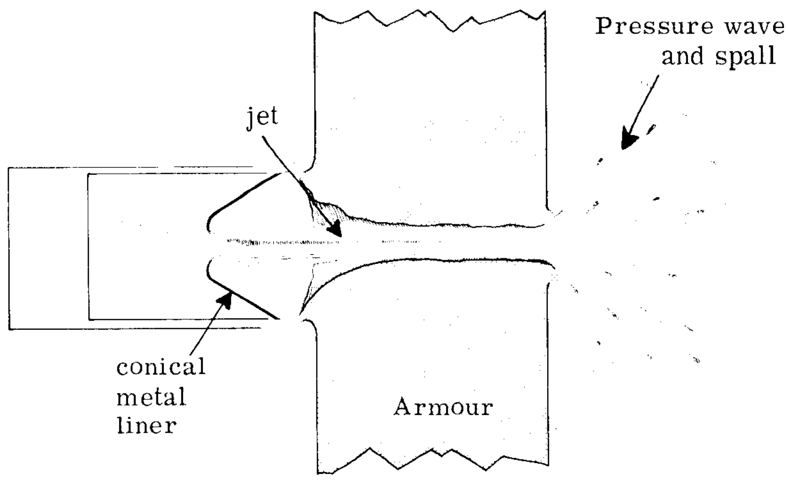


Fig. 5.10 HEAT effect

Whilst penetration is important, it is not the main consideration: damage behind the plate is the most important effect to achieve. HEAT causes damage behind the plate in three ways; it can be with the jet itself, with the spall, and by physiological and pyschological effects against the crew by pressure, temperature and flash. The jet will, having penetrated, disable anything in its path, but as it is so narrow its chance of hitting anything inside the tank is comparatively small. The main lethality effect comes from the spall. The wider the exit hole on the inside of the tank, the more spall will be formed and the greater will be the lethality. HEAT lethality is therefore often assessed on the basis of exit hole diameter. Lethality, however, is obtained at the expense of penetration and vice versa. The narrower the jet, the greater the penetration, but the less the lethality. On the other hand, the wider the jet, the greater the lethality but the less the penetration. The HEAT warhead designer must compromise between these conflicting factors.

A number of factors influence the lethality amongst which are cone diameter, material and thickness of liner, and stand-off distance. A good discussion of these factors may be found in (1). Overall, HEAT is capable of achieving considerable depths of penetration in armoured plate and is not affected by spaced armour. There is however, one drawback arising from the use of the shaped charge shell with a conventional gun. It is the creation of spin imparted from the rifling. The general effect of this rotation is to diffuse the jet over a larger area, scooping out wider, shallower depressions in the target surface, rather than the punching of deep, narrow holes. As the rate of rotation is increased the penetration may be reduced by as much as 50%. Consequently, the easiest solution is to fire a hollow charge projectile from a smooth bore launcher or gun. Often this is impossible because a rifled gun is required for other types of ammunition, so a slipping driving band can be employed; this is the British solution for tank guns. In terms of target effect, compared to other chemical modes of attack, HEAT is efficient and economic in its use of high explosives, and therefore particularly applicable to lightweight anti-tank weapon systems.

High Explosive Squash Head (HESH) Effect

Another method of attacking armour is HESH, or High Explosive Plastic (HEP) as it is known to the Americans. Because it employs a considerable amount of explosive, it has a considerable secondary use against bunkers, buildings and personnel.

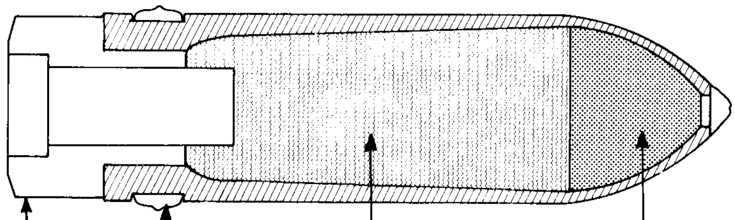


Fig. 5.11 Basic components for a HESH projectile

In a HESH projectile HE is placed in contact with the armour, instead of being separated by a 'stand-off' gap. It is base fuzed; it is thin-walled so that it can squash against its target; and the nose contains a pad of inert composition which tops the HE filling. When such a projectile strikes a hard surface, such as armour or concrete, the thin shell walls collapse and the HE contact pancakes on to the surface. The fuze then detonates the filling and, because it is now in direct contact with the target, a high velocity compressive stress wave is propagated through it. When this reaches the rear of the plate, the wave is reflected back through the plate as a tension wave. The rebounding tension wave meets further waves coming in the other direction. They combine, setting up a reinforced stress wave which exceeds the tensile strength of the plate. As a result a large scab is torn off the rear of the plate to act as a projectile inside the attacked vehicle.

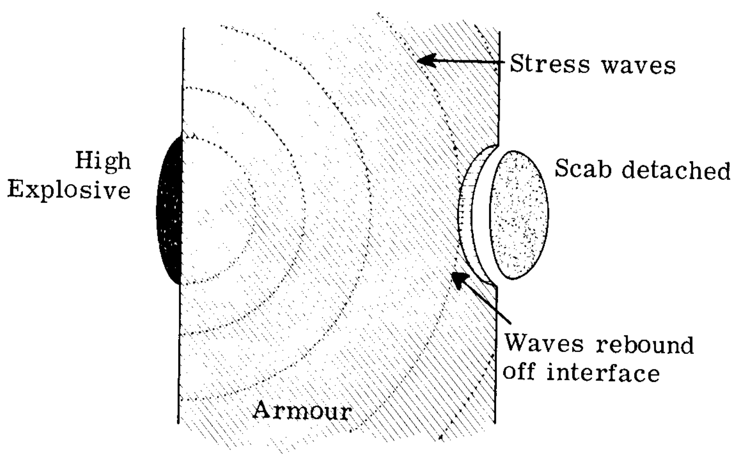


Fig. 5.12 HESH effect

There are a number of practical considerations, most of which are discussed in (1). The presence of a pad of inert material in the nose of the projectile serves to cushion the HE filling against the action of the impact. This avoids a premature detonation, which would spoil the pancake effect and result in a relatively harmless burst like that of a conventional HE shell. The most serious limitation of HESH is that the effect is entirely defeated by spaced armour on the target. Also, a HESH projectile striking at about 700 m/s will pancake on any plate thicker than 6 to 8 mm; but its inherent kinetic energy will result in its penetration of anything less. On the credit side, HESH is largely unaffected by the angle of attack, and in fact by striking at an oblique angle, the effective area of contact may be increased. The optimum penetration performance is actually attained when the angle of attack is about 40°.

Despite its limitations HESH is particularly useful against many types of target such as bridge supports and concrete buildings. It comprises a very effective armour defeating projectile although it suffers from a relatively low velocity and higher trajectory. Comparing instances when HEAT and HESH fail to defeat, say, tank armour, HESH may still have a sufficient physiological effect upon the crew to put the vehicle out of action.

To make the armoured vehicle designer's task as difficult as possible, it is important that kinetic energy, HEAT and HESH attack capabilities are kept. This prevents the development of armour which can defeat particular types of attack. For example, it would be comparatively easy to develop lightweight spaced armour to defeat HESH, but it would be vulnerable to kinetic energy attack. The converse is also true.

MATHEMATICAL APPROACH

Until this point in the chapter a descriptive approach has been followed. The following section illustrates the various types of approaches that may be taken to solve actual problems. Those readers not interested in the solution of terminal ballistic problems should miss out the section.

Terminal Ballistics — Part II

THEORY OF TERMINAL BALLISTICS

Approach

The theory of terminal ballistics is relatively new compared to the theory of internal and external ballistics. The techniques of investigation for impact on solid targets consist primarily of empirical relations (based on experiments), analytical models, and computer modelling.

Empirical Relations

Empirical relations used in penetration mechanics are most useful when the number of variables being correlated is small. The empirical relations commonly used in penetration mechanics are based on experimental correlations of such parameters as penetration depth P, crater volume C, the ballistic limit V_{50}*, with the dimensions of the projectile, mass m, the impact velocity V, and obliquity θ and the type of target and its thickness h. Examples are:

$P/d = a_0 \, mV^2/d^3$	Fundamental Armour Equation
$P/d = a_1 \, (mV^2/d)^{0.6993}$	Milne-de-Marre
$P/d = 2mV^2/a_2\pi \; \pi d^3$	Morin
$P/d = a_3\rho_t \; \ln\,(1 + a_4V^2)$	Dideon
$E_o = a_5 \, d^{1.5} \, h^{1.4}$	Marre
$C = a_6 \, (1/2mV)$	Helie

*The ballistic limit V_{50} is defined as that velocity which penetrates for 50% of the time.

In these equations d and E are the diameter and perforation energy of the penetrator, ρ_t is the target density and a_i are arbitrary constants. The above relations avoid the problem of incorporating a large number of variables by restricting the correlation to a single material; in this way the parameters of the material description are reduced to a minimum or eliminated. When used with strict adherence to this limitation, the empirical relation may be the best available prediction. It shows that penetration mechanics is a complicated phenomenon that depends on many parameters. However, empirical relations do not provide any insight into the physical processes described, so we turn towards other methods.

Analytical Models

Analytical models provide correlations similar to those from empirical studies, but they also include relations between parameters of the system on the basis of physical requirements. Simple representations may combine an empirical element with physical principles: for example, the resistive force can be assigned an empirical formulation. The empirical force law is then used to determine a correlation of some penetration parameter involving velocity, V, by solving an equation of motion. The best known of these laws is the expression

$$F = a_7 + a_8 V + a_9 V^2$$

Usually, in order to solve the penetration problem simply, the application of the physical principles must be accompanied by restrictive assumptions. Examples are ideal rigidity of the projectile, ideal plastic behaviour of the target and specified modes of deformation in the target that lead to perforation. Momentum or energy balances between striker and target have been successfully applied. Other formulations have included: 1. target compression due to projectile motion, the effect of friction and target inertia, 2. only target resistance and inertia, and 3. the combination of target inertia, resistance to flow and friction.

Hypervelocities

The larger stresses occurring in hypervelocity impact permit neglect of rigidity and compressibility of the striking bodies and the impact is viewed as fluid flow. The description of material properties is then greatly simplified. Subsequently, fluid mechanics can be used with correction terms for target and projectile strength. Fluid models have been used to study the penetration of shaped charges, and to study wave propagation in long rods simulating kinetic energy penetrators.

Computer Modelling - Numerical Solutions

Because the theory of terminal ballistics is so complicated, most of the current work uses numerical schemes. Briefly, we list some of the current computer codes that are used to solve particular problems usually depending on the target size. Goldsmith, (2), provides extensive references for further study. For thin plate penetration and perforation, there are various computer codes covering this rather large area of terminal ballistics. A computer solution for the deformation

of clamped circular plates under explosive loading, using the structural elastic/plastic program DEPROSS, has been found to compare satisfactorily with measured deflections and strains (3). STEEP (4) was employed to study the ejecta characteristics behind thin plates of aluminium, copper and cadmium struck by spheres at 7500 and 15000 m/s. The expansion of the debris cloud was traced and the mass axial momentum and kinetic energy of the ejecta was evaluated. CRAM (5) computed perforation problems such as plugging and petalling failure. CRAM was also used for the study of penetration of equal weight, equal base diameter steel cylinders, truncated cones, and ogival projectiles into steel and aluminium targets at velocities up to 1000 m/s. The monograph on High Velocity Impact Phenomena (6) contains several papers on the subject as well as a description of the CAMEO calculations for the behaviour of debris generated upon hypervelocity perforation of thin targets and shields. Reference (7) contains a section on penetration mechanics with several useful references to numerical aspects of thin plate perforation. The impact of blunt and pointed tungsten carbide and steel projectiles on steel armour and aluminium plates at velocities of 1000 to 1300 m/s was examined by the SHEP code. Penetration, plugging and hole formation were explicitly treated (8). Applications to a numerical study of hypervelocity impact, plugging failure in thin plates occurring during ballistic impact and shaped charge jet formation is described in (9).

Thick Targets

Many of the numerical procedures already described either apply equally well or may be readily converted to the case of thick targets, although here, the failure processes are far more complicated.

Semi-Infinite Targets

Numerical methods have been applied to soil penetrations using the codes WAVE-L and PISCES-DL2 (10). The results indicated that after the non-deforming projectile has been fully embedded in a given soil layer, the penetration process is steady. Disturbances of the soil do not extend far from the projectile but there is generally separation of the soil from the projectile along the nose. Deformations of the projectile are extremely small so that the projectile is assumed rigid. Earth penetrators are intended to deliver a warhead whose detonation either damages a subterranean structure or else causes extensive cratering to a runway or road etc.

SUMMARY

The ranges of impact velocity, thickness, angle of attack, and material characteristics are the basis for establishing some order among the approaches to the subject of penetration mechanics. There is a great diversity among these approaches, which vary in sophistication from simple descriptions of test results to detailed mechanical theories. The diversity of these approaches is due in part to several aspects of the contributing phenomena. The range of impact speeds result in mechanical phenomena whose descriptions are nonlinear and involve significant changes of material behaviour.

The simplest approach for a specific problem is to derive an empirical relationship, but this has the obvious disadvantage of non-applicability to other situations. Analytical models are potentially the means for arriving at a simple solution because these derive conclusions from fundamental concepts through idealisations and simplifications and put the expressions in a tractable form. Numerical solutions of sets of differential equations that represent the fundamental conservation laws and constitutive relations certainly promise to provide the answers sought, but these have disadvantages of expense and complexity.

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SELF TEST QUESTIONS

QUESTION 1 What are the main factors that affect impact phenomena?

Answer ....

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QUESTION 2 What is the difference between penetration and perforation?

Answer ....

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QUESTION 3 What three phases generally describe the interaction of a long-rod penetrator with a target?

Answer ....

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QUESTION 4 Mention some ways in which a target may be defeated by a projectile without completing its passage through the armour.

Answer ....

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QUESTION 5 What are the main features of an APDS round?

Answer ....

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QUESTION 6 What are the advantages of an APFSDS round over an APDS round?

Answer ....

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QUESTION 7 Under what conditions would
(a) blast be effective?
(b) fragmentation be effective?

Answer (a) ....

....

(b) ....

....

QUESTION 8 How does a HEAT missile achieve its main effectiveness against armour?

Answer ....

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QUESTION 9 What type of anti-tank projectile would be relatively ineffective against spaced armour?

Answer ....

QUESTION 10 What are the main ways of investigating the subject of terminal ballistics?

Answer ....

....

ANSWERS ON PAGE 204

6.

Wound Ballistics

THE PROBLEM

Wound ballistics may be defined as the study of the motion of missiles within the body and the wounding capacity of them. It is therefore basic to the understanding of the wounding effects of both bullets and fragments, which in the past have caused over 90 per cent of combat casualties. A wound results from the absorption of energy imparted by a missile when it strikes and penetrates tissue: consequently the study of wound ballistics requires knowledge of the behaviour of the bullet in flight and the effect it has on the tissues it penetrates. These effects depend on the size, shape, composition and above all, the velocity and stability of the bullet. In the body, the elasticity and density are the most important factors which influence the retardation of the penetrating missile. The soft tissues of the body, like water, are 800 to 900 times as dense as air and when a bullet hits these tissues it nearly always becomes unstable; any angle of yaw that is present will be greatly increased, with a subsequent increase in damage. In fact trials have shown that a very slight yaw at impact may develop yaw within the target of between 50 to 100 degrees. Generally, a bullet can cause injury in one of three ways, depending on its velocity.

CAUSES OF INJURY

Laceration and Crushing

The main effect of low velocity, that is subsonic, missiles is to penetrate the tissues and crush and force them apart. Only those tissues that have been directly hit are damaged and the actual wound damage is comparable to that of a knife wound. The laceration and crushing effect is not generally serious unless vital organs or major blood vessels are directly injured.

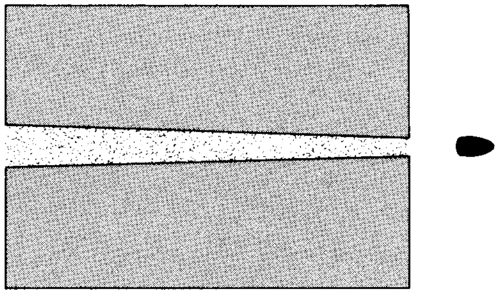


Fig. 6.1 Wound track of subsonic projectile

Figure 6.1 demonstrates the typical path of a small subsonic projectile penetrating a gelatine block.* The bullet simply gives up its energy by creating a small wound track formed by the displacement of the gelatine. The two other types of injury occur with high velocity or supersonic projectiles.

Stress Waves

When a high velocity bullet forces a wound track through solid tissue, it compresses the medium in front of it and this region of compression moves away as a stress wave of spherical form (see Fig. 6.2). The velocity of this stress wave is approximately equal to that of the velocity of sound in water, that is, 1500 m/s. The changes of pressure due to stress waves only lasts about a millionth of a second, but they may reach peak values of up to 100 atm.+ Although the inertia of the tissue does not allow the stress wave to bring about any transfer of tissue, damage to the nerves, for example, may be caused at a considerable distance from the permanent wound track.

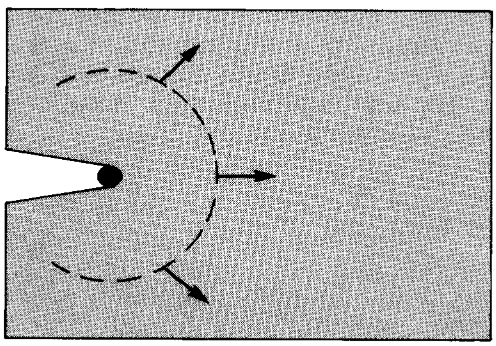


Fig. 6.2 Stress wave formed by high velocity projectile

*20% gelatine and soap are often used in wound ballistic studies for simulating the human body.

+1 atm 0.1 MPa

Temporary Cavitation

The severe wounding effects caused by high velocity projectiles are mainly due to temporary cavitation. When the bullet enters the body, momentum is transferred to the surrounding tissues. This momentum causes the tissues to move and oscillate even after the passage of the projectile and thus a large cavity is created: it is often approximately 30-40 times the diameter of the missile. This cavity, in the space of a few milliseconds, goes through several pulsations, expanding and retracting before reverting to a semi-permanent shape. These violent changes in the tissues are sufficient to fracture bones, rupture organs and blood vessels and damage nerves outside the immediate path of the projectile. The shape of this temporary cavity is, except where yaw has taken effect, in the form of an ellipsoid. Because the cavity has a subatmospheric pressure and is connected to the outside by entry and exit holes, bacteria from the outside, together with clothing and debris, are actively sucked into the depth of the wound.

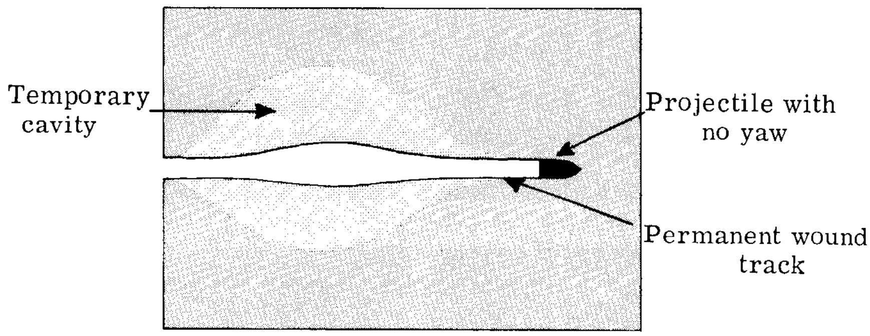


Fig. 6.3 Temporary cavity formed by high velocity projectile

Cavitation takes place mainly after the passage of the missile, and accounts for the explosive nature of high velocity missile wounds. The greater the energy that is imparted to the tissues, the greater is the size of the temporary cavity and the more extensive the damage.

AMMUNITION DESIGN AND WOUND EFFECTIVENESS

Logical Limitations

The design of projectiles has a great influence on the wounding effects. The Hague Convention of 1899 banned bullets which have jackets with slits or an opening at the point which would permit the jacket to strip upon impact with the target. Such a design would allow the lead core to mushroom, causing very serious entrance wounds. However, the Convention lacked a full appreciation of such factors as yaw and velocity effects. This is because would ballistics as a science had not begun to develop.

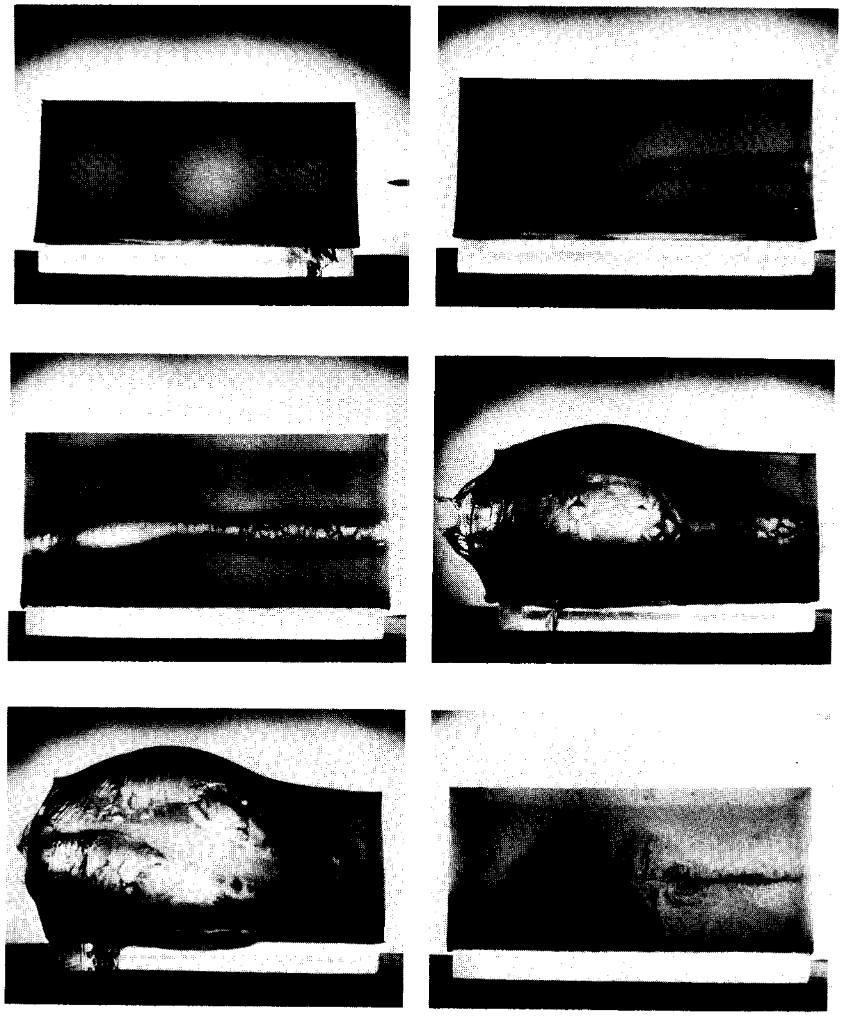


Fig. 6.4 Selected stills from a high speed cine film showing the temporary cavitation

Velocity and Stability

Pistol bullets have a relatively low amount of energy available to cause damage. All rifle bullets have much more energy and if a rifle bullet is stable on impact, it might go right through the target giving up only 10-20 per cent of its energy. If it leaves unstable, it may give up 60-70 per cent of its energy, and hence cause a more severe wound. In the extreme, if the bullet fragments on impact, all the energy will be used up and a severe wound would result. The external appearance of a bullet wound can be deceptive. If the bullet enters or leaves the body end-on, then it will commonly have a small hole, irrespective of the severe damage it may have caused during the passage through the body. If the bullet enters,

or more commonly, leaves the skin sideways on to some degree, then the hole in the skin will be large and ragged, but the internal damage may be no more severe.

Projectile Design

For the same amount of total energy expended, the design of the bullet can make profound difference in its effect on tissues. For example, a soft bullet will flatten on impact, producing a much greater surface area and, therefore, greater retardation. Such a design will produce a very early release of energy (see Fig. 6.5 (a)). If the bullet is an unstable jacketed bullet, the energy would be more rapidly released as the bullet begins to yaw, Fig. 6.5 (b). If it is a stable jacketed bullet, the energy is dissipated rather late and a longer wound track is observed, Fig. 6.5 (c). Obviously the length of the wound track is a measure of the stability of the bullet in soft tissue.

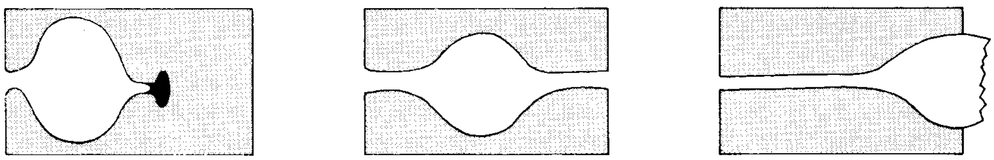
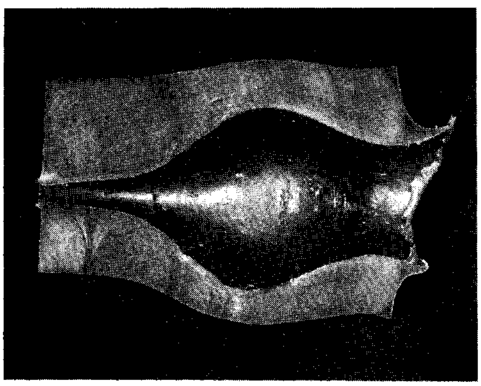
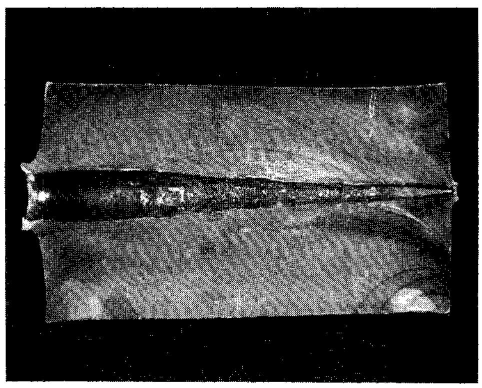


Fig. 6.5 The respective wounding effects of a soft bullet, unstable jacketed bullet and stable jacketed bullet



Cavity produced by a 6 mm steel sphere travelling from left to right with an impact velocity of 1100 mls.



Cavity produced by a 7.62 mm NATO ball round travelling from left to right with an impact velocity of 850 m/s.

Fig. 6.6 The permanent record of cavitation effects as shown in standard soap blocks

Finally, two particular instances of the effect of design on wounding is worth mentioning. The British .303 inch Ball Mark 7 created an unintentionally severe wound. In this design the nose filling consisted of a metal plug. The nose tended to break off and could cause in a wound, break up of the bullet with consequent serious wounding effects. The intention behind this design however had simply been to improve the external ballistics of the bullet by placing the centre of gravity as far back as possible. Other projectiles such as the 6.5 mm Japanese rifle round had a thickened lead core at the base, which despite its small calibre and lowish velocity, tumbled badly upon hitting tissue, and caused serious wounds. Therefore, in recent years, bullet design has been concerned not only with external ballistics and accuracy, but also with the wounding effects.

BODY ARMOUR

The Need

The idea of body armour has been with man since the earliest of times. In modern life there is a requirement for lightweight armours capable of stopping high velocity shot. This requirement covers both battlefield and internal security situations. Early bullet-proof vests were made mainly from metal plates, but they were generally clumsy. However, advances in metallurgy and synthetic materials have made it possible to produce protective coverings which are effective but at the same time light enough to be worn without discomfort.

The capacity of a high velocity projectile for causing bodily injury or damage, on impact, is determined by its kinetic energy E. More accurately, for projectiles of different shapes, it is convenient to define damage capacity as E/A where A is the contact area on impact. The armour used must resist penetration of the projectile, and also dissipate most of the kinetic energy of the shot. For high velocity bullets this is extremely difficult to achieve within the severe constraints of weight and bulk. Also, because the body is in close contact with the armour, and the vital organs are susceptible to relatively small amounts of kinetic energy, the problem is further complicated. Differing solutions to the problem have been produced.

Metallic Armour

Metallic armour acts by totally rejecting the bullet and preventing any penetration. It usually breaks up a projectile and distributes the energy over a large surface area of the plate. Metallic armours are the best material for stopping high velocity rifle bullets, and it is the only material successful in stopping steel-cored bullets or armour-piercing projectiles. Modern techniques can produce a light low-alloy steel armour which is used in light jackets for total ballistic protection and is also put into doors and seats of vehicles and helicopters. Also, metallic armours can usually survive more than one bullet impact in the same area. The ability to withstand such close and repeated shots is something that other armours do not have and is a strong factor in choosing metal for certain applications. On the other hand, all metallic armours suffer from the

disadvantage of spalling on projectile impact: this is a result of shock wave reflection from the back surface. Another disadvantage of metallic armour, illustrated in Fig. 6.7, is that if the plate is in direct contact with the body a shock wave can be transmitted to the underlying tissues and damage from cavitation can occur.

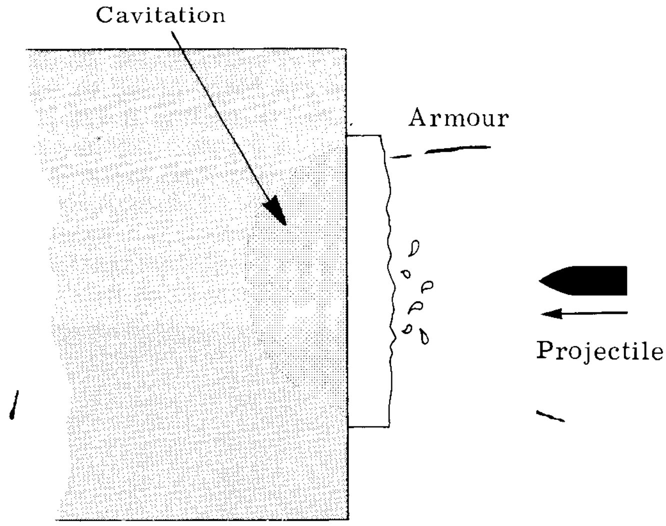


Fig. 6.7 Possible cavitation effects with metallic armour

Reinforced Plastic Armour

Reinforced plastic armour protects by partially rejecting the bullet and partially absorbing it. In the process each bullet strike partly destroys the area it hits. Plastic is therefore not as good as metal at defending against more than one hit over a small area and the area of armour which has been struck must be replaced if the overall protection of the plate is to be maintained. Moulding the plastic provides easy shaping for a particular application. In acceptable weights it offers protection only against low-velocity bullets although it is particularly good against blast and fragmentation attack from grenades or shells.

Ceramic Armour

Ceramics are characterised by their hardness and low density. Armour-piercing bullet design favours high density, high hardness core materials, in particular tungsten carbide. For the effective shatter of tungsten carbide bullets, the ceramic layer should exceed the hardness of the bullet, so something like boron carbide is typically used for a ceramic armour. Ballistic attack is defeated by the material starring, cracking and generally degenerating rapidly. Any one area cannot accept more than one hit, and to limit the effects of this it is generally used in the form of small tiles, each one of which will absorb one hit. It needs to be backed with some extra form of protection such as textile armours, but does offer very high levels of protection from bullets of all normal calibres. The

major disadvantage of ceramic armours is that they are expensive. It is also necessary to replace any damaged tiles which makes this type of armour suitable only for specialised uses.

Textile Armour

Textile armours are a series of layers of heavy-weave cloth such as nylon or Kevlar sewn together. They will stop any round nosed bullet, for example about 16 layers or plys will stop a 9 mm bullet travelling at 330 m/s (see Fig. 6.8). The bullet is caught in the heavy weave as it hits, and the nose is rapidly deformed into a rounded mushroom. This mushroomed bullet is then unable to force its way through the mesh of fibres and stops. The material, despite the number of layers, is flexible and can be easily tailored into a waistcoat. There is a possibility of injury to the wearer, due to the cavitation that occurs upon impact, but the extent of any injury is generally considered to be relatively small.

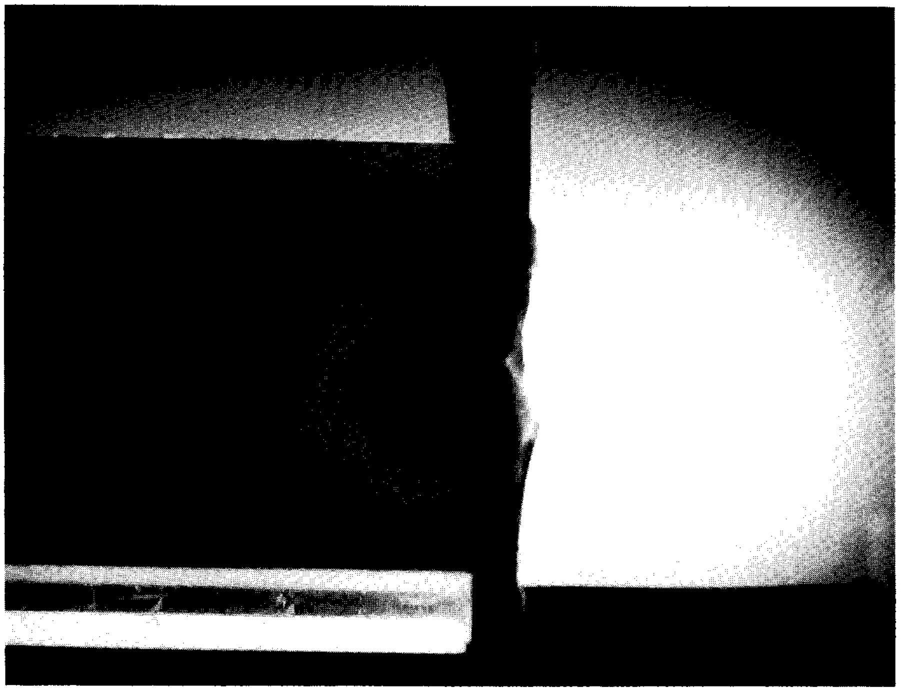


Fig. 6.8 The non-penetrating impact of a 9 mm bullet travelling at 330 m/s against 16 plys of Kevlar body armour

Transparent Armour

Transparent armours are generally made from one of three materials; glass, polycarbonate or acrylic plastic. With all of these some degree of lamination is necessary to break up the impact of the projectile's stress waves, and prevent sudden shattering. Glass armour is the only transparent armour which can be made to withstand rifle bullets, but there are obvious serious drawbacks to its

use: the weight, and the need for lamination behind the glass to prevent damaging splinters flying off with each impact are the most important. Plastic armour, such as polycarbonate sheets, is virtually unbreakable and makes a useful riot shield. In thick laminations it offers good protection against low-velocity bullets, but not against rifle calibres. It also has the advantage that it generally does not crack.

Composite Armour

The concept of composite armour is to dissipate a large amount of kinetic energy without transmitting it to the target situated behind the armour. This may be achieved in three ways: firstly, it slows the projectile to reduce the kinetic energy; secondly, it induces the projectile to break-up and decrease the kinetic energy per unit area, and thirdly it absorbs residual kinetic energy.

In practice a triple composite armour consisting of an outer layer of metal, an inner layer of ceramic and a backing layer of plastic material provides an effective shield. The metal layer slows down the projectile while the ceramic layer induces shatter of the projectile and finally the plastic layer, since it is capable of elastic deformation, absorbs the kinetic energy. In addition, the plastic provides a protective backing and by a suitable choice may minimise stress reflections and hence spalling. Such a composite armour offers significant advantages over metallic armour. At 60 kgm^2 it will protect against a 7.62 mm shot, as compared with over 100 kgm^2 required for steel armour. Composite armour also offers a better performance than metals for varying angles of attack. However, although composite armour use the advantages of the individual armours from which it is constructed, it also suffers from various disadvantages: it has a limited re-use capability because the progressive breakdown of the ceramic layer is not easily repairable; in addition it is expensive to manufacture.

Composite armours are now replacing the other types of armour in many situations. This is due to the comparative lightness and effectiveness of the ceramic layer. Although the processing of ceramics is expensive, new technology will probably reduce the cost. In the end, however, the particular type of armour used ultimately depends upon the likely form of attack.

SELF TEST QUESTIONS

QUESTION 1 What physical quantity causes a wound to result?

Answer ....

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QUESTION 2 What factors influence the retardation of a penetrating missile?

Answer ....

QUESTION 3 What causes temporary cavitation?

Answer ....

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QUESTION 4 Is it necessarily true that projectiles with the same kinetic energy create the same wounding effects?

Answer ....

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QUESTION 5 What are the main requirements of body armour?

Answer ....

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QUESTION 6 What are the advantages and disadvantages of metallic armour over other types of armour?

Answer ....

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QUESTION 7 What are the advantages and disadvantages of ceramic armour?

Answer . . . .

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QUESTION 8 How does textile armour prevent penetration?

Answer . . . .

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ANSWERS ON PAGE 205

7.

Ballistics Instrumentation

INTRODUCTION

All ballistic phenomena occur at high speeds, and a great number of complex interactions occur; to study these phenomena it is necessary to use specialised measuring equipment. In this chapter we shall look at various high speed photographic techniques applicable to all branches of ballistics, followed by external and in-bore instrumentation. We are usually concerned with microsecond photography, projectile velocity, displacement, yaw and spin, as well as pressure measurements up to several thousand mega pascals.*

HIGH SPEED PHOTOGRAPHY IN BALLISTICS

The Problem

High speed photography is used directly in all branches of ballistics. As ballistic photography is such a wide ranging subject we shall only present those techniques which are well established and capable of easy use.+ In general, all events in ballistics take place quickly, and in order to obtain clear pictures exposures are usually very short. As object movement becomes faster, exposure must decrease and illumination must increase or the image must be artificially enhanced electronically.

Photographic System

A photographic system consists primarily of a light source, optics, a shutter and

*1 mega pascal is equivalent to about ten atmospheres of pressure.

+Fuller (1) gives a more detailed account of high speed photography in ballistics and many further references.

a means of recording the photographic image. The particular type of photographic system chosen depends upon the duration of the event. An indication of typical time durations for various illuminating sources is as follows:

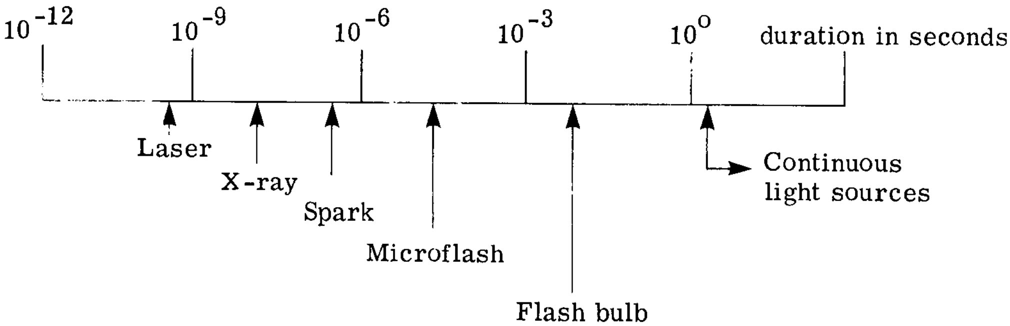

Fig. 7.1 Time scale for various illuminating sources

For example, the displacement of a 7.62 mm projectile travelling at 840 m/s is approximately 1 mm in 1.2×10^{-6} seconds. To "freeze" such a motion requires a light source which has a duration considerably less than this, and so we use a spark. If we were to use a microflash system which has a typical time duration flash of 10^{-5} seconds, the bullet would be blurred. If a single event is to be recorded then a continuous light source with a rapid shutter can be used in conjunction with stationary film; if however a time history is required it is usual to use either high speed motion film or to have fixed film with opto-mechanical scanning of the image across the film or synchronised with a moving film. For a limited number of images a sequence of sparks or flashes can be recorded in superposition on a stationary film.

Microflash Photography

With the microflash system an intense flash of a few microseconds duration is produced when a high voltage is discharged through a gas-filled lamp normally containing argon or xenon, and this flash can be used to provide both the illumination and the short exposure necessary for a sharply defined image of a rapidly moving object, as illustrated in Fig. 7.2.

Fig. 7.2 A microflash photograph of a 105 mm shell in flight

Fig. 7.3 A microflash photograph of 120 mm A.P.D.S. in flight. The blurring is caused by long duration of flash

In general this technique is only suitable for subsonic projectiles. This is evidenced by the blur on Fig. 7.3. It is worthwhile comparing this photograph with Fig. 7.12 using Ballistic Synchro. Microflash photography has applications in ballistics work for the study of projectiles in flight to observe an irregularity in behaviour or damage that might be sustained during the firing process. A series of photographs of a particular event can be obtained by using multiple flashes. The same system may also be used to measure the spin of projectiles. To achieve this lines are painted longitudinally on the shell body which is photographed in flight in the light of two microflash lamps spaced a short distance apart down the range. The time interval between the two flashes is recorded and the rotation of the shell during this interval can be measured from the photograph: from the measurements the rate of spin can be calculated. Triggering the flash at the required moment may be performed with the breaking of a light beam, an electrical circuit, or by sound detection.

Spark Photography - Shadowgraphs

Sparks are certainly the oldest form of illumination for ballistic photography and their popularity and use continues. The great advantage of this method lies in its short exposure time. The spark, which is usually less than 0.5 microseconds in duration, is produced by discharging a high voltage through a gas such as argon on the opposite side of the projectile from the camera; and the light emitted from the spark then 'shadows' the object. When obtaining a shadowgraph of a supersonic projectile the spark is often triggered by a detector sensing the nose shock wave from the projectile. The shock wave is seen as a shadow because the large pressure disturbances generated by the shock wave create density gradients in the shock wave which refract the light away from the direct path, as evidenced by the appearance of darkened lines.

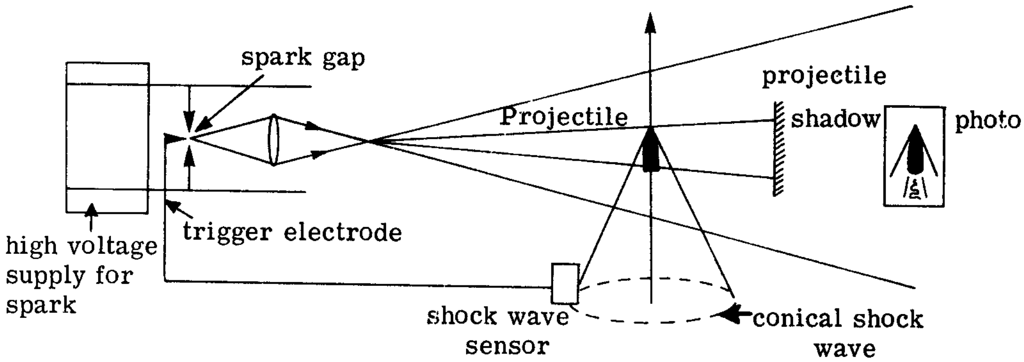

Fig. 7.4 Experimental arrangement for spark photography

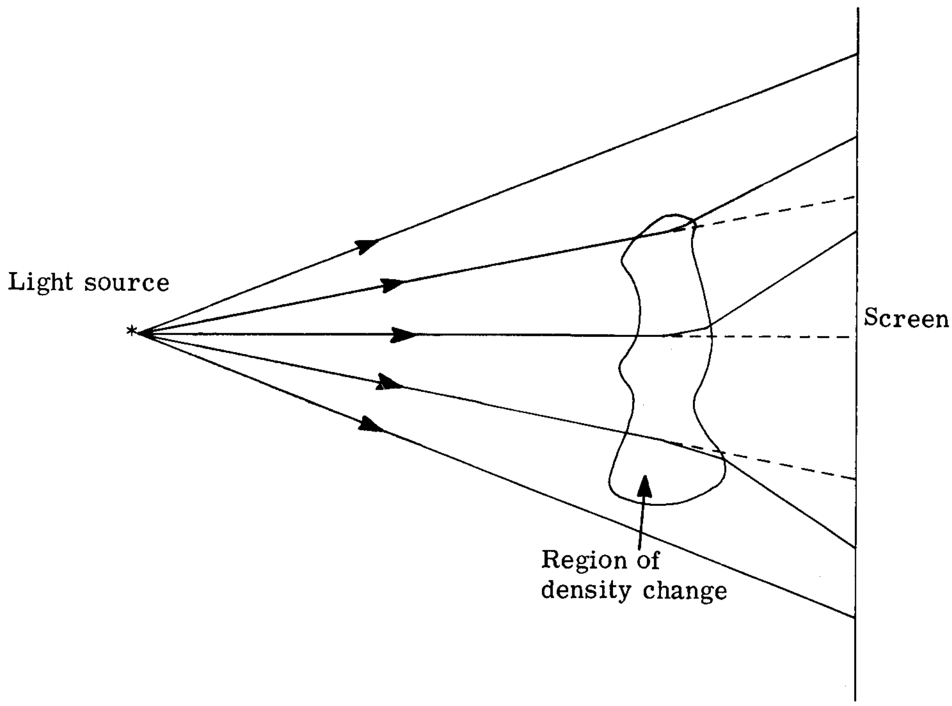

Fig. 7.5 Diagrammatic representation of the shadowgraph

By triggering several sparks after known time delays successive images may be recorded on the film.

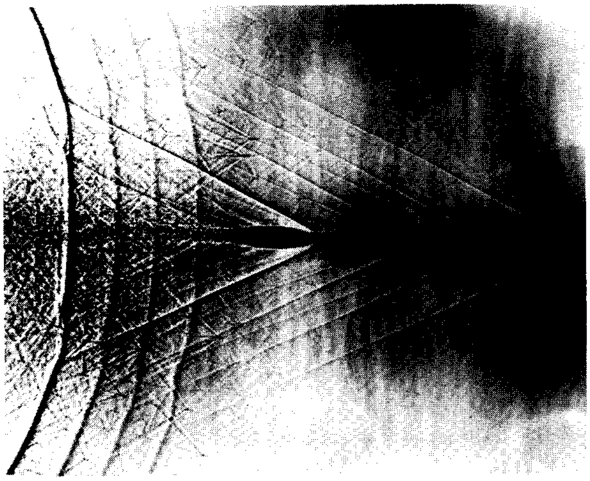

Fig. 7.6 Multiple spark photograph

Spark photography is used for studies of armour protection and penetration, yaw measurement, air resistance, aerodynamic performance of a particular shape of projectile, and surrounding airflow around an event when it is subject to a pressure front.

Schlieren

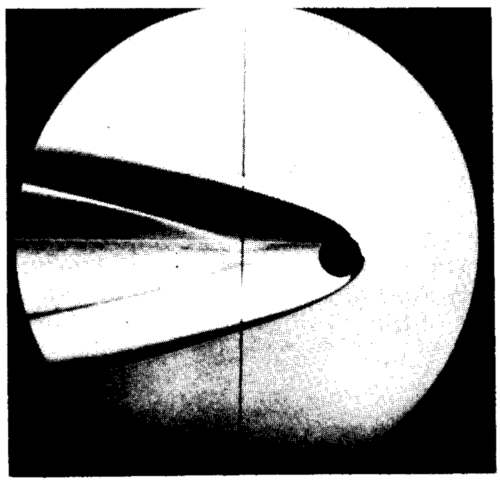

Fig. 7.7 Schlieren photograph of sphere

The Schlieren technique is another method for detecting the density change in the field of view. All of the light emitted from a small light source is focused on to a small obstruction; only the light refracted by the pressure disturbance in the field of view is focused on the photographic plate.

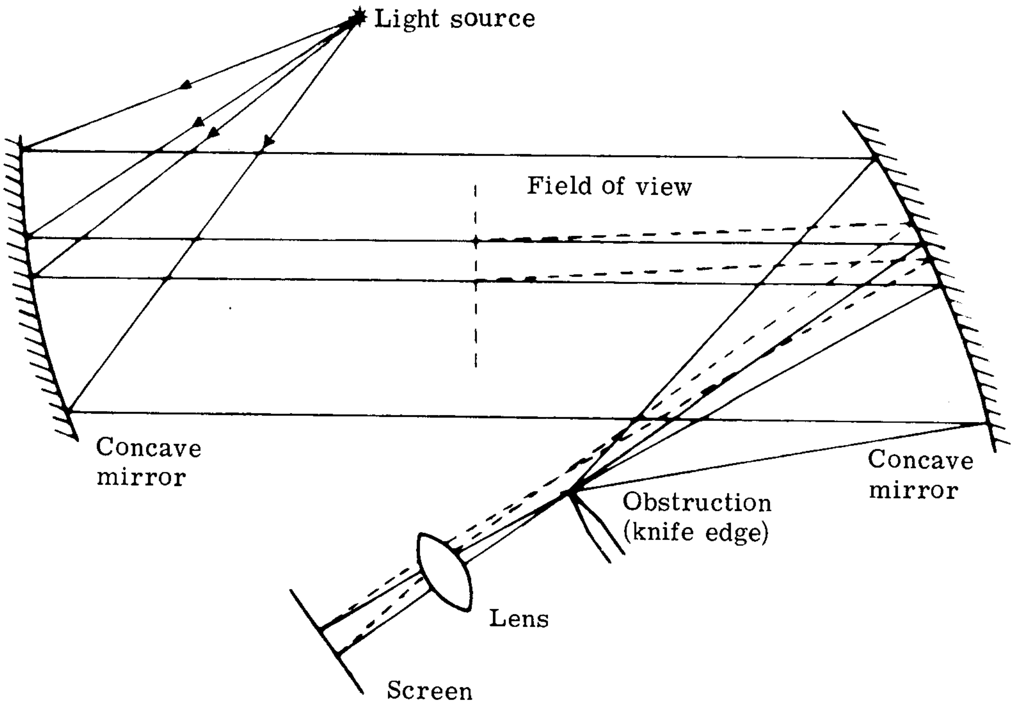

Fig. 7.8 The Schlieren system

HIGH SPEED CINE PHOTOGRAPHY

Slow Speed Cine

By filming at a high framing rate and projecting at the normal 16 or 24 pictures per second (p. p. s) the time scale is extended and a recorded event can be viewed in slow motion. Conventional cine cameras employ a mechanical system of film transport where claws engage in perforations along the edge of the film and pull it past the aperture frame by frame. The movement is intermittent because the film is held stationary in front of the aperture during exposure. The result is a sharp image.

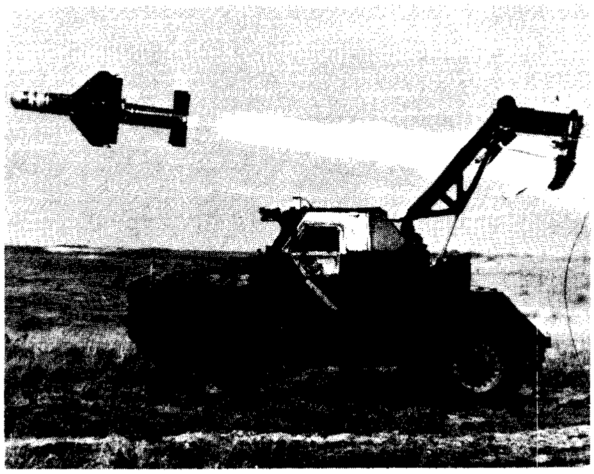

Fig. 7.9 Rocket launching

Figure 7.9 shows a rocket launch taken at a frame rate of 40 p.p.s. with an exposure time of 1/1000th sec. The filming speed of the conventional type of camera is limited ultimately by the mechanical strength of the film base; above 1000 p.p.s. the film will tear. Hence, to achieve higher framing speeds a different system of film transport is required. This is provided by the continuous running type of camera.

Medium Speed Cine

A medium speed camera gives up to about 10,000 full p.p.s. with 16 mm film and over 20,000 p.p.s. with quarter frame. It is extremely versatile and probably the most widely used in ballistics work. Here the film is run through the camera continuously at high speed and is not held stationary during exposure. To prevent blurring of the image due to film movement, an 'optical compensator' is incorporated between the lens and the film; illustrated in Fig. 7.10. This takes the form of a rotating glass block which is usually geared to the film drive. As the block rotates, light rays from the lens are refracted to an extent which depends upon the angular position of the block and the effect is to move the image in the same direction that the film is travelling. Definition is not quite as good as that produced by the conventional camera. Continuous running cameras of this type can operate satisfactorily at filming speeds up to 10^4 p.p.s. If a standard 30 m film spool is accelerated to this rate, the film will reach a speed of 273 km/h and be fully exposed in less than three quarters of a second. Correct triggering of the event is therefore critical.

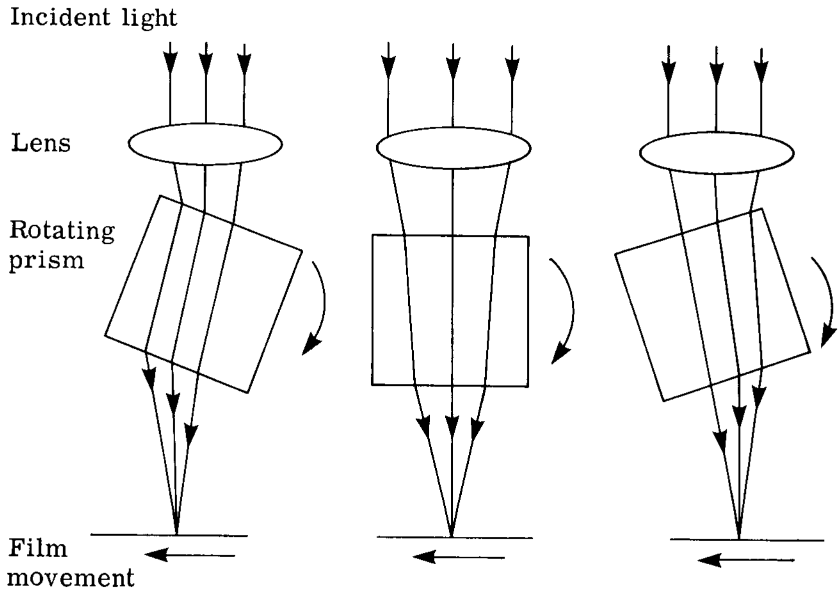

Fig. 7.10 Compensation for film movement by rotating glass block

The framing rate is controlled by the motors driving the sprocketed drive wheel and take-up spool. Useful recording time is governed by the time used in accelerating the camera to operating speed. It is common practice to record a time base on the film edge to allow accurate film speed determination during analysis. This is achieved by a light emitting diode which simply flashes a small light spot on the edge of the running film at a known rate, for example at every 1/1000th sec.

Lighting

The very short exposure time associated with fast filming speeds makes high intensity lighting necessary. For most purposes suitable lighting can be provided by photoflood lamps. Typically, these are rated at 750 or 1000 watts each and have built-in reflectors so that the light can be focused onto the subject. A more sophisticated form of lighting is known as 'synchronous flash' or 'strobe illumination' which is produced by a stroboscopic unit. Briefly, this provides a high intensity repetitive flash of short duration by means of a gas discharge lamp. It can be synchronised to the camera so that it flashes at the same rate as the filming speed; that is, a separate flash is produced for each exposure. The available flash energy provides a more intense illumination than from photoflood lamps.

Ballistic Synchro

Ballistic synchro is a powerful technique using the medium speed camera without its compensating block in a streak mode. This technique produces high quality front-lit photographs showing projectile condition, yaw and spin rate, as well as

the operation of sabots and obturators. The method used to compensate for projectile motion is shown in Fig. 7.11.

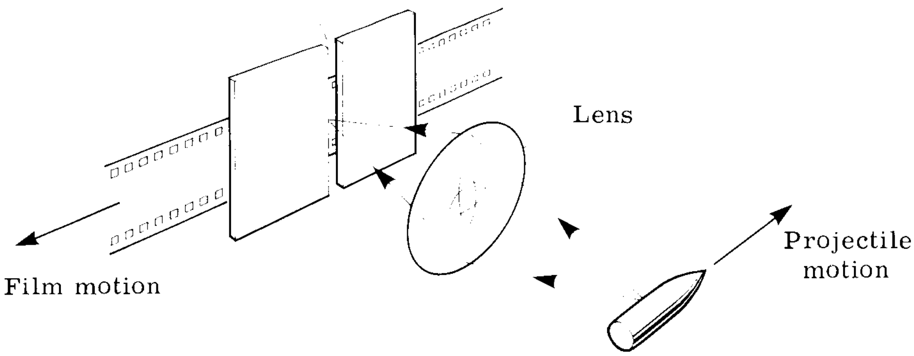

Fig. 7.11 Ballistic synchro technique

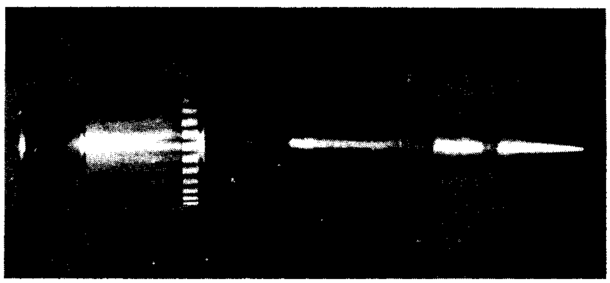

Fig. 7.12 A ballistic synchro record of 120 mm A.P.D.S.

High Speed Cameras

Beyond a certain acceleration rate, film fails mechanically and higher framing rates must be achieved by alternative means. There are a number of types of high speed camera. Many allow the film to remain stationary and sweep the image across it by reflection from a revolving mirror or prism. They are constructed in two forms, either as streak or framing cameras. Streak photography is obtained when the camera is used without any film motion compensation at all. It is particularly applicable to explosive studies: the record is taken as a continuous exposure as the film passes the lens. Figures 7.13 and 7.14 show the two types of camera arrangements. They are capable of a rate of several million p.p.s. A large amount of light is required to illuminate the subject, and problems may occur due to the limited number of frames per film length. The problem of event and mirror position synchronisation may also occur.

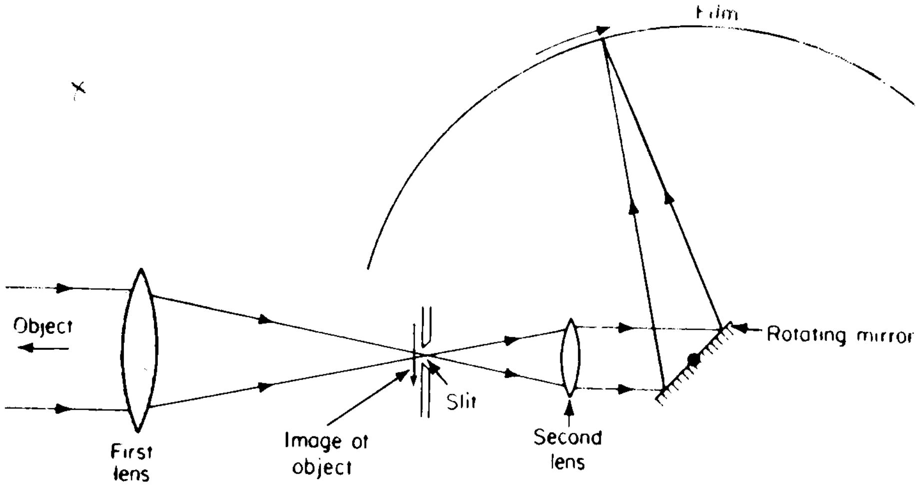

Fig. 7.13 The rotating mirror streak camera

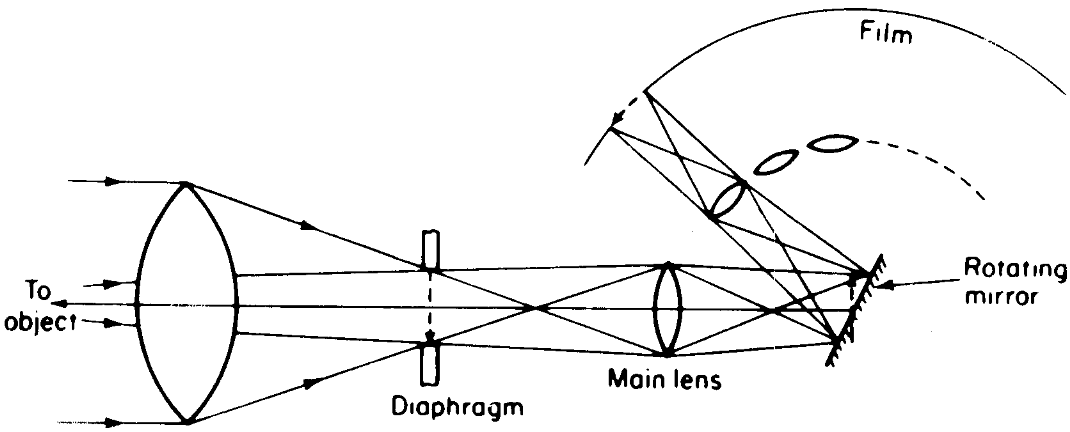

Fig. 7.14 The rotating mirror framing camera

Image Converter Cameras

Image converter cameras have two main advantages: they can electronically enhance the brightness of an observed event and as a result can give very short exposure pictures on ordinary film. Framing speeds of a typical Image Converter type camera can range from 10,000 to 2 x 10^7 p.p.s. with exposure times down to 10ns.* Figure 7.15 shows the internal layout of such a camera.

*1 ns = 1 x 10^{-9} secs.

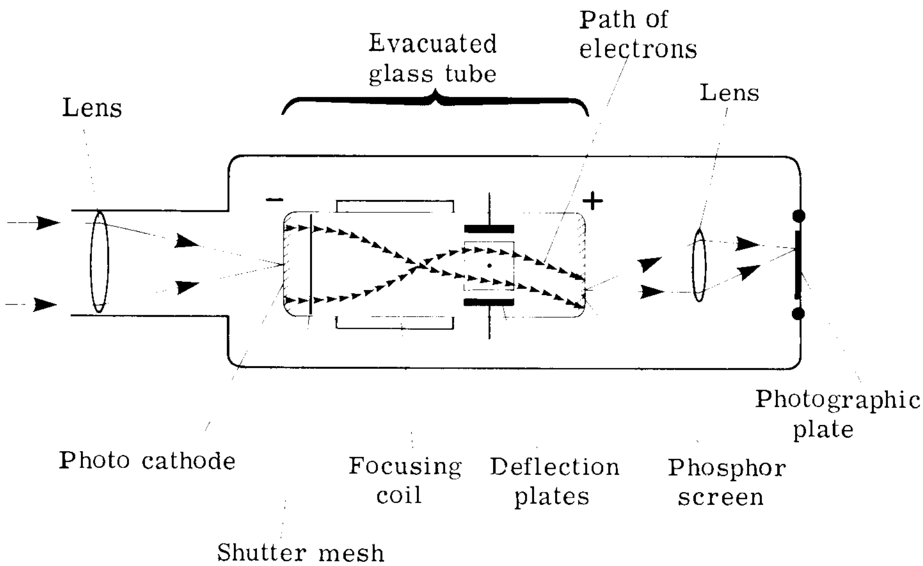

Fig. 7.15 Image converter camera schematic

The event is imaged onto a photo-cathode and the stimulated electron beam is focused onto a phosphor screen. In passing through the image intensifier the original brightness of the image becomes enhanced to the point where a conventional film camera can photograph it from the phosphor screen. By electronic control the electron beam carrying the image can be shuttered and scanned across the phosphur to enable multiple consecutive images to be separately recorded. Figures 7.16 and 7.17 show respectively the types of image converter photographs taken in normal mode and streak mode.

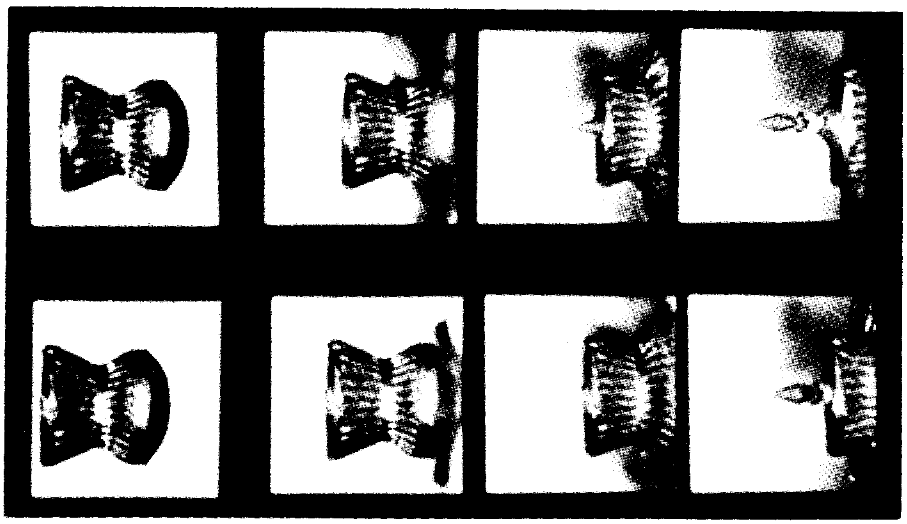

Fig. 7.16 Lead pellet impacting on hard target - Framing rate = 1×10^5 p.p.s.

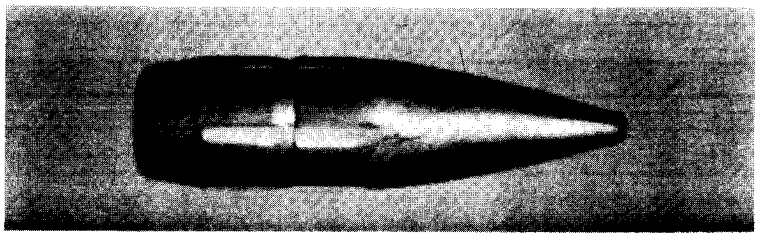

Fig. 7.17 Streak photograph of 7.62 mm bullet travelling at approximately 840 m/s

Flash Radiography

In some ballistic applications the only way to obtain suitable pictures is by using X-rays, which are produced by electron bombardment of a metal anode. The wavelengths normally used for flash X-ray photography range from 10^{-4} to 10^{-1} nano-metre with durations of 1 microsecond or less.

Fig. 7.18 X-ray shadowgraph of 0.45 automatic during firing

Initially only single exposures could be obtained, but developments have led to repeated discharges in a single tube giving up to 150-200 p.p.s. Currently, framing rates of 10^6 and higher have become possible. Up to five pictures may be recorded on the same film frame, but this leads to loss of definition; for higher framing rates a continuous film is more common. A recent development has been the use of image intensifiers where the X-ray image is changed into an optical image which can then be photographed by an ordinary cine camera.

Radiographs of internal functions in guns (see Fig. 7.18) require high energies in order to achieve penetration. Typical measurements may be the observation of projectile movement in the bore with time or the behaviour of parts of the internal mechanisms under firing stresses. The region just beyond the muzzle where the projectile is moving through the expanding propellant gas is a particularly difficult region for photographic observations due to the large amount of flash; so it is a suitable subject for the radiograph technique, as is the mechanism of sabot detachment and integrity of the projectile itself. By photographing against known datum lines the projectile yaw can also be seen.

During the external ballistics phase of the flight conventional photography is usually more effective. However, X-rays can be used to study the condition and operation of internal mechanisms such as fuzes. In terminal ballistics many target materials of great interest are metal or other materials which are opaque to normal photographic methods. Flash radiography thus offers a unique method of observing the interactive process between projectile and target during penetration. Another advantage of radiography is the ability to photograph through the luminosity often associated with impact. Radiography is also useful in wound ballistics, where conventional photography is of course limited.

EXTERNAL BALLISTICS MEASUREMENTS

In this section some of the more common forms of measuring equipment suitable for the study of external ballistics are described. As previously mentioned, we are usually concerned with the displacement of projectiles, their velocity, retardation, yaw and spin rate.

Photodetector Counter Chronometer (PCC)

PCC equipment is an opto-electronic system for measuring the velocity of a projectile by timing its trajectory very accurately over a known distance. It consists of two light intensity detector screens and a velocity computing counter with interconnecting cables. Different types of detector screens are available, depending upon the size of the projectile and the angle of elevation at which they are fired. The detector screen comprises a lens, slit and photodetector coupled to an amplifier: it is arranged so that light from the sky immediately above the detector screen is collected by the lens and focused on to the slit through which it passes to the photodetector below. The passage of the projectile interrupts some of the light reaching the photodetector causing a sudden reduction in photodetector current which triggers the counter.

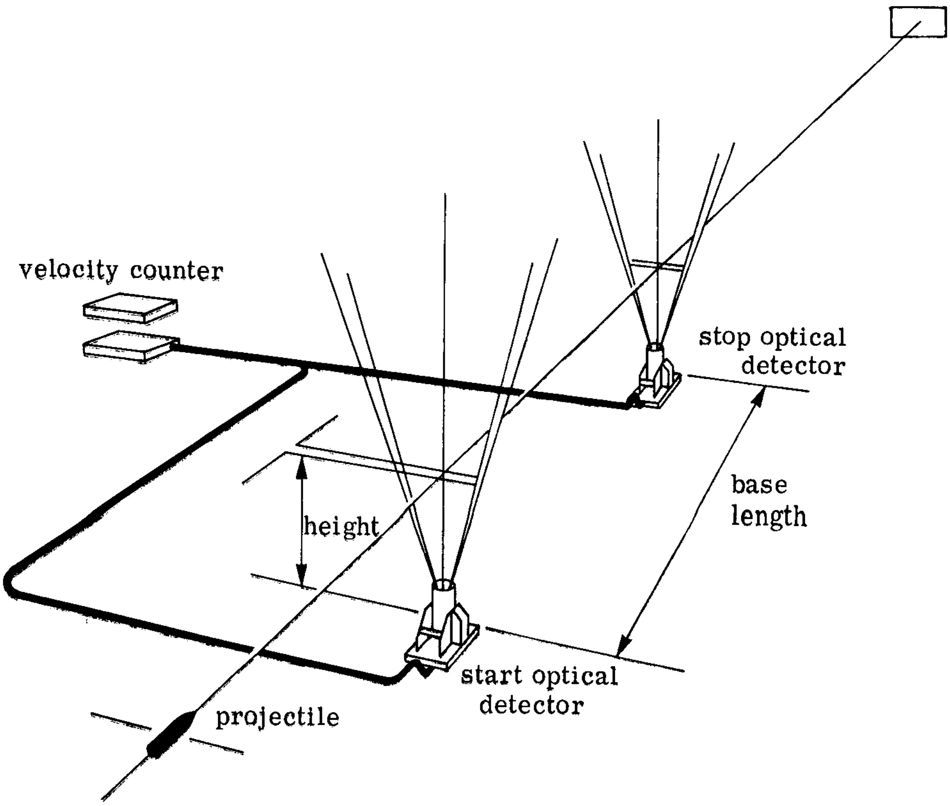

Fig. 7.19 PCC equipment

To measure velocity the projectile is fired over two detector screens placed a known distance apart. The pulse from the first detector screen starts the counter chronometer, and the pulse from the second one stops it. As the distance between the screens is known, the velocity can be measured by the velocity computing counter. On large ranges shells may be fired over many photoelectric detectors spaced at known intervals. The time taken by the shell to reach each of the detector screens in turn is measured by electronic counters and a computer can convert these results to provide a print-out of retardation at corresponding velocities down range. Since retardation is markedly dependent on any yaw that occurs during flight, photography may be used to record the attitude of the shell simultaneously.

Principle of Radio-Doppler

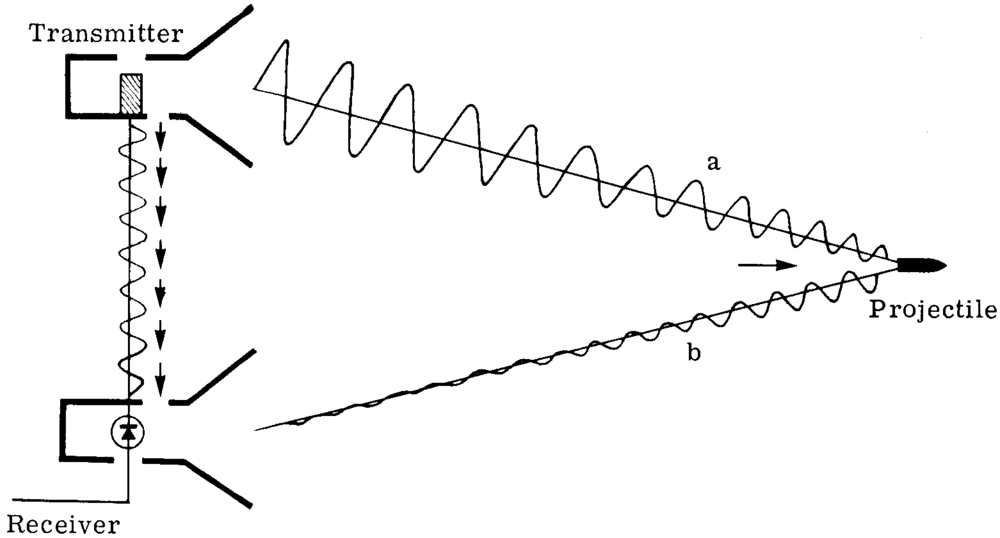

Fig. 7.20 Principle of radio-doppler

If a radio transmitter emitting continuous waves is directed towards the base of a shell moving directly away from it, the base of the shell will reflect some of the radiation. The return signal can then be picked up by a receiver located near the transmitter. If this receiver is also fed directly from the transmitter it will simultaneously receive two sets of waves which will interfere, one set direct and one set via the shell. For certain positions of the shell along the trajectory the two sets of waves will be in phase and will reinforce each other so that the receiver will record a maximum signal; for intermediate positions of the shell the two waves will arrive out of phase and a minimum signal will result. Consequently the receiver will show a series of maxima and minima as the shell recedes from the emitter. The total length of the waveform is the sum of the transmitted and reflected waves, that is, a + b. If the shell moves a distance of one wavelength λ, the total length of the two waves becomes $a + b + \lambda = (a + \lambda/2) + (b + \lambda/2)$. Therefore, the time between each maxima is the time taken for the shell to travel half a wavelength ($\lambda/2$). Using the relation speed = distance / time, we have

$$\text{time between maxima} = \lambda/2V$$

where V is the velocity of the shell. Thus the number of maxima per second, which is the so-called Doppler frequency f_D will be given by:

$$f_D = 2V/\lambda$$

Since the wavelength λ is known, a measurement of f_D gives a measurement of the velocity of the projectile V over a large part of the trajectory. Most commercial

radio doppler units also provide facilities to produce a retardation record from the range of velocities measured. In general the radio-doppler system possesses advantages over a PCC system because it can be used in adverse weather conditions such as fog or thick mist which would defeat an optical method; neither does it require any apparatus to be set up forward of the gun. Typically it has the disadvantages that a sufficiently large signal is not reflected from the base of a small-calibre projectile to allow its use in that case and it is necessary to have a fairly good idea of the expected velocity. However, neither of these disadvantages apply in the case of the calibration of guns in the field because it is usual to already have a knowledge of the approximate size and velocity of the projectile.

Yaw Sonde

The yaw sonde is the only satisfactory means at present available for measuring yaw and spin behaviour of shells in free flight throughout most of the trajectory and for all angles of fire. Although different versions are used in the United Kingdom and North America, the principles are basically the same. The equipment consists essentially of a miniature radio transmitter which is frequency modulated by a solar cell when the cell is periodically illuminated by the sun's rays passing through a pin-hole. The solar cell is 'V' shaped and mounted parallel to the shell's axis (see Fig. 7.21). It is located at right angles to a pin-hole in the casing of the shell. The equipment is fitted in a package having the same shape and characteristics as the nose cone of the shell which it replaces and must be capable of withstanding the typical 30,000G force that is experienced on firing.

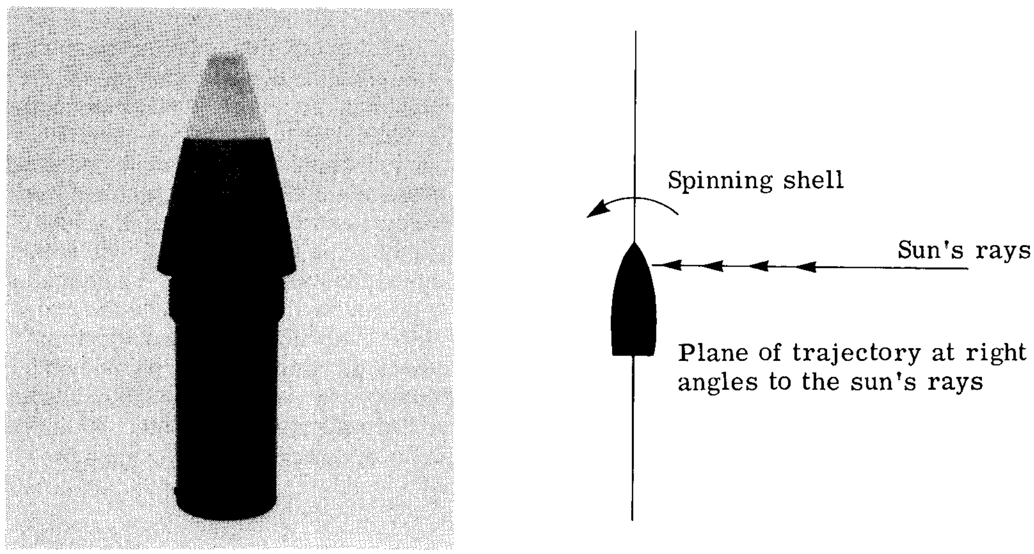

Fig. 7.21 Principle of yaw sonde

The shell is fired in a plane at right angles to the sun's rays. In flight, the image of the sun, formed by the pin-hole, sweeps across the 'V' shape solar cell, crossing first one arm of the 'V' and then the other, during each revolution of the shell. The position at which the sun's image sweeps across the 'V' cell arms

depends on the angle between the axis of the shell and the sector to the sun. Figure 7.22 illustrates the principle of the 'V' cell.

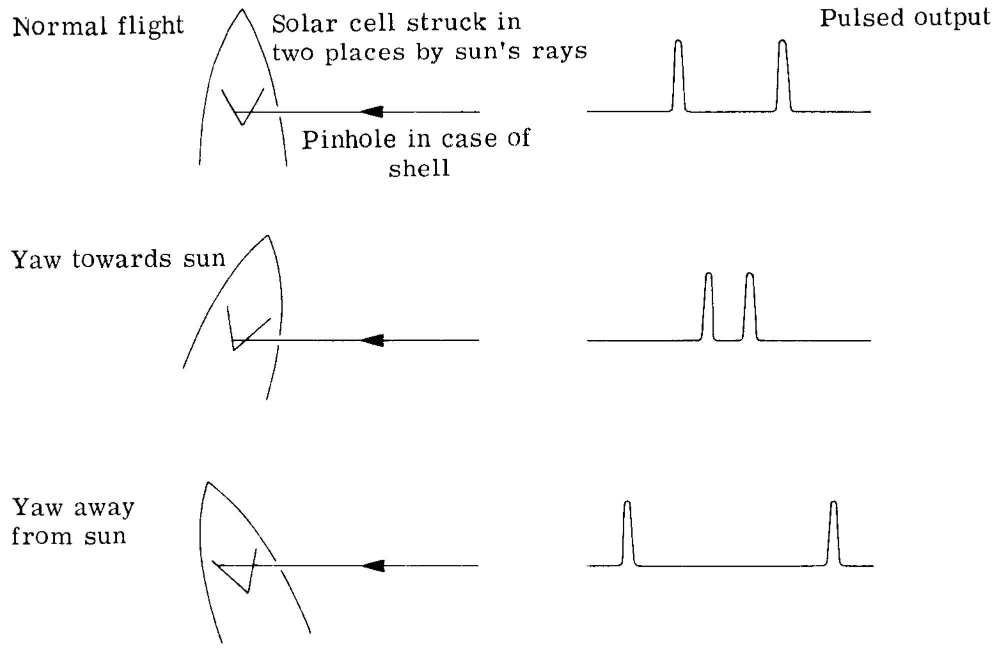

Fig. 7.22 Principle of the 'V' cell

Transmission of the information from the sonde starts approximately 0.4 seconds after firing and is picked up by a ground receiver. The output consists of a train of pulse pairs, one pair having been transmitted for each revolution throughout the shell's trajectory. The time between successive pulse pairs defines the spin rate, the time of each pair defines the angle of yaw.

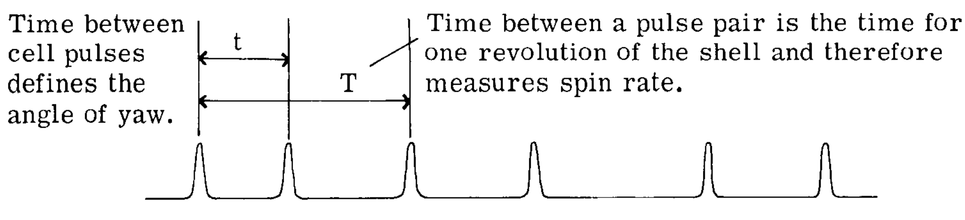

Fig. 7.23 Interpretation of pulse record

The information can be stored on magnetic tape and used to calculate yaw, spin rate, precession and stability. A typical example is shown below in Fig. 7.24.

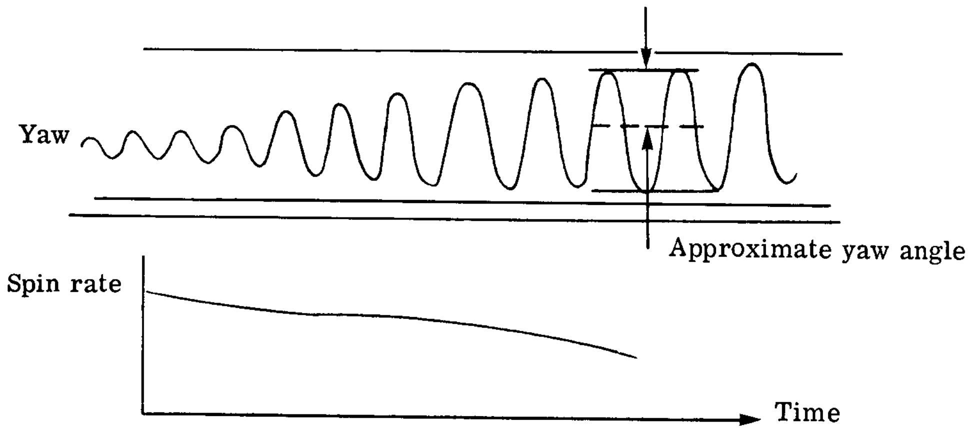

Fig. 7.24 Results from yaw sonde

Shot Position Indicator

The shot position indicator is an electronic target which automatically records the X and Y co-ordinates of a supersonic projectile as it passes through the sensor plane.

The system can be used for measuring the position of a single shot, or the positions of the individual shots in a burst of automatic fire; it will work over a range of calibres from 4 mm to greater than 30 mm. The co-ordinates of the projectile are found by measuring the point of impact of the wave front on two orthogonally mounted sensor rods. If the projectile passes normally through the target plane, the shock wave front expands as a circle centred on the position of the projectile.

The two points of impact of the shock wave front on the two axes are then related to the X and Y co-ordinates of the projectile (see Fig. 7.26). The point is found by measuring the difference in arrival time between longitudinal vibrations at each end of the rod. The shock wave at the impact point sets up a system of longitudinal and transverse waves in the rod material. The longitudinal waves travel faster than the transverse waves and so reach the end of the rod first. Two transducers mounted on the ends of the rod convert the longitudinal waves to two electrical signals; by measuring the difference in arrival times of those signals the point of impact l may be found from $l = (t_1 - t_2)\,V/2$ where V is

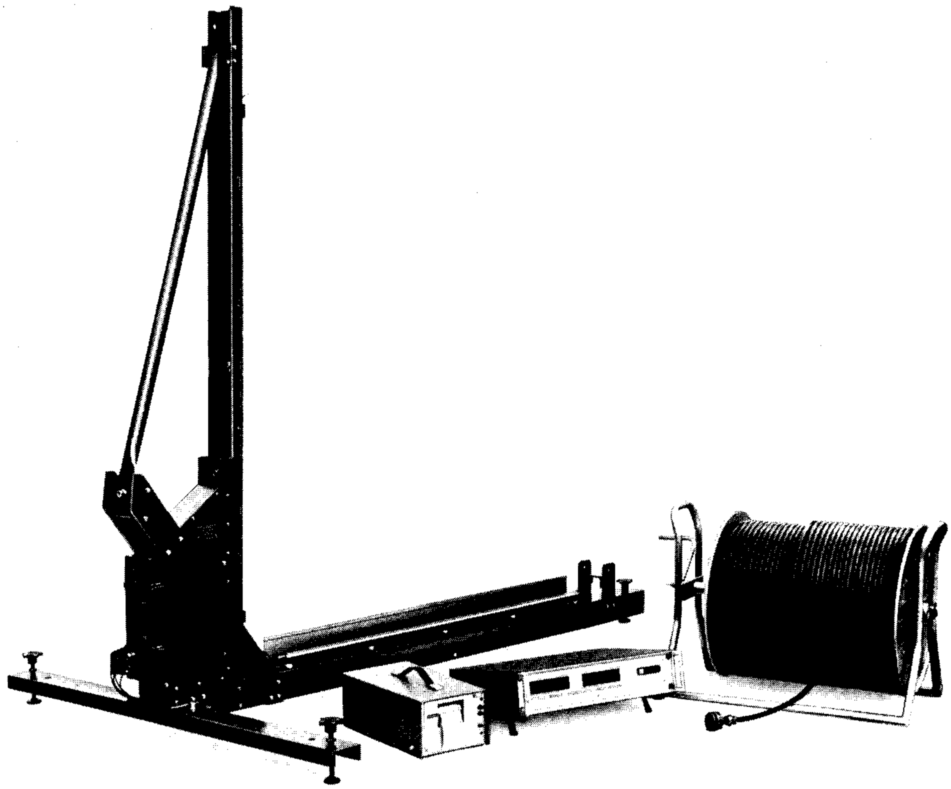

Fig. 7.25 Shot position indicator

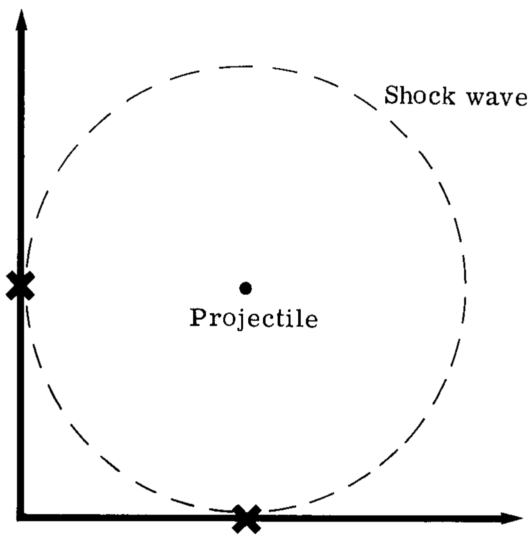

Fig. 7.26 Position of shock wave front

the velocity of the longitudinal waves in the rod material. When the shock wave strikes the middle of the rod, the two arrival times are equal and will be zero. This implies that the origin of the co-ordinate system is at the centre of the rod and that a positive or negative sign must be used to distinguish readings from either side of the centre point. The sign convention follows the normal Cartesian co-ordinate system, illustrated in Fig. 7.27.

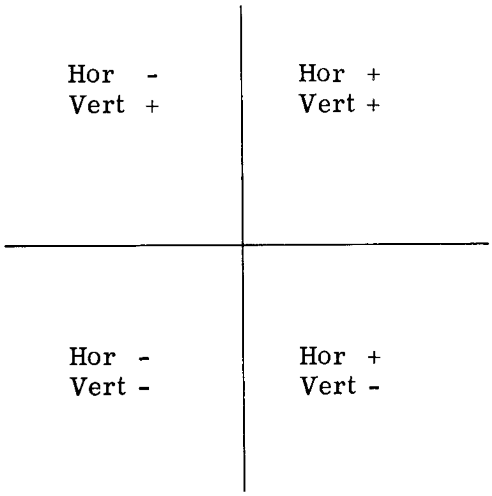

Fig. 7.27 Cartesian co-ordinate system

IN-BORE PRESSURE INSTRUMENTATION

The Problem

In-bore instrumentation consists of the determination of the loading and response of projectiles and projectile components during in-bore travel. In-bore travel is defined as beginning at the time of projectile ramming (shot-start) and continues through the interior ballistic cycle and muzzle exit until the projectile is no longer influenced by the propellant gases. There are two prime objectives: the first is to quantify the specific loading and response as actually experienced by the projectile and gun; the second is to accurately simulate these loading conditions accurately in mechanical devices other than guns for both cheaper testing and for establishing models. The principal measurements required are pressure and projectile travel; the latter leads to projectile velocity and acceleration. If the measuring equipment is carried in the projectile then it must withstand large forces due to large accelerations, both longitudinally and transversely, because of vibration in the bore during travel. Additionally, the information must be extracted from the projectile, and this requires either flexible wire connections or radio telemetry. It is clear that gun-mounted rather than projectile mounted instrumentation provides an easier environment. Some of the currently available

measuring techniques used in internal ballistic instrumentation are described in the following paragraphs.

Gas Pressure Measurement

Two basic types of pressure gauge are used. There are gauges which only provide a record of the maximum gas pressure reached during the firing process, for example, the Crusher gauge, and then there are gauges such as the Piezoelectric gauge, which provide a continuous record of pressure/time variation.

Two types of crusher gauge are in common use, one fitted flush with the inside of the gun barrel wall and the other which is located in the propellant for large guns.

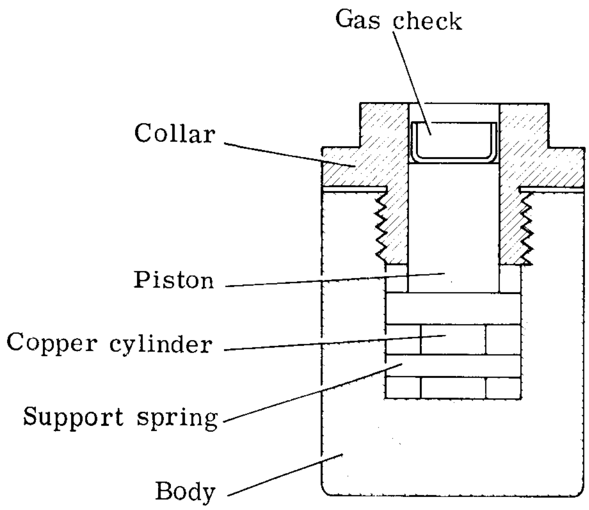

Fig. 7.28 Crusher gauge

An assessment of the maximum pressure reached in the gun barrel and chamber during the firing process is obtained by subjecting to this pressure a cylinder or sphere of copper and measuring the extent to which it is permanently compressed or crushed. A typical example would be a copper ball of 4.76 mm diameter. The crusher gauge response represents only about 80% of peak pressure but it can be calibrated against the piezoelectric gauge to read full peak pressure. Although not an extremely accurate gauge it is adequate for making comparative measurements.

The piezoelectric gauge is again usually placed flush with the interior wall of the barrel. The gauge uses the phenomenon that certain crystals, when subjected to pressure, develop an electric charge proportional to the applied pressure. This charge is generated at an unusually high impedance, in the range of 10^9 - 10^{14} ohms: the cause is the high impedance connecting cables which are necessary to

prevent loss of signal charge. The charge is fed into a charge amplifier to convert to a lower impedance and also to produce a voltage proportional to the input charge. The signal can then be recorded. Recordings are commonly made into digital storage oscilloscopes or on to magnetic tape and subsequently processed.

IN-BORE PROJECTILE MOVEMENT

The Problem

Methods are required to provide information on the variation of axial displacement of a projectile in a gun during firing.

Microwave Interferometer

In a microwave interferometer, a microwave source of known frequency produces a signal which is fed to an antenna. The microwave energy is directed by a cheap replaceable reflector so that it propagates down the barrel of the gun. Some of the incident signal is reflected from the moving shell and picked up by the antenna. When the instantaneous received and transmitted signals are compared there is a cyclic change in phase difference which can be calibrated in terms of the projectile motion. Various commercial microwave bands are used according to the size of gun barrel and to the delineation of motion that is required: 'X' Band (10,000 MHz) and 'Q' Band (35,000 MHz) are the most common. They give interference maxima every 15 mm and 4.5 mm respectively when used in their fundamental modes. The number of maxima is therefore a measure of the total distance traversed.

The only continuous method for measuring in-bore velocity is to use the interferometer, where the frequency of the output signal from the interferometer is proportional to shell velocity. This frequency shift, known as the Doppler Shift, can be used to evaluate shell velocity.

Laser Interferometry

Because of its shorter wavelength the laser provides superior discrimination to that obtained from a microwave system. However, because the frequency shift associated with even low velocity movement becomes impracticably high it is necessary to provide automatic electronic processing to deal with the very large numbers of interference maxima. The use of a CO_2 laser in preference to the more commonly available helium-neon laser increases the distance between interference maxima from 0.315 μm to 5.3 μm: this enables projectile displacement to be made with greater accuracy than with the 'Q' Band microwave source where the distance between maxima is 4.5 mm.

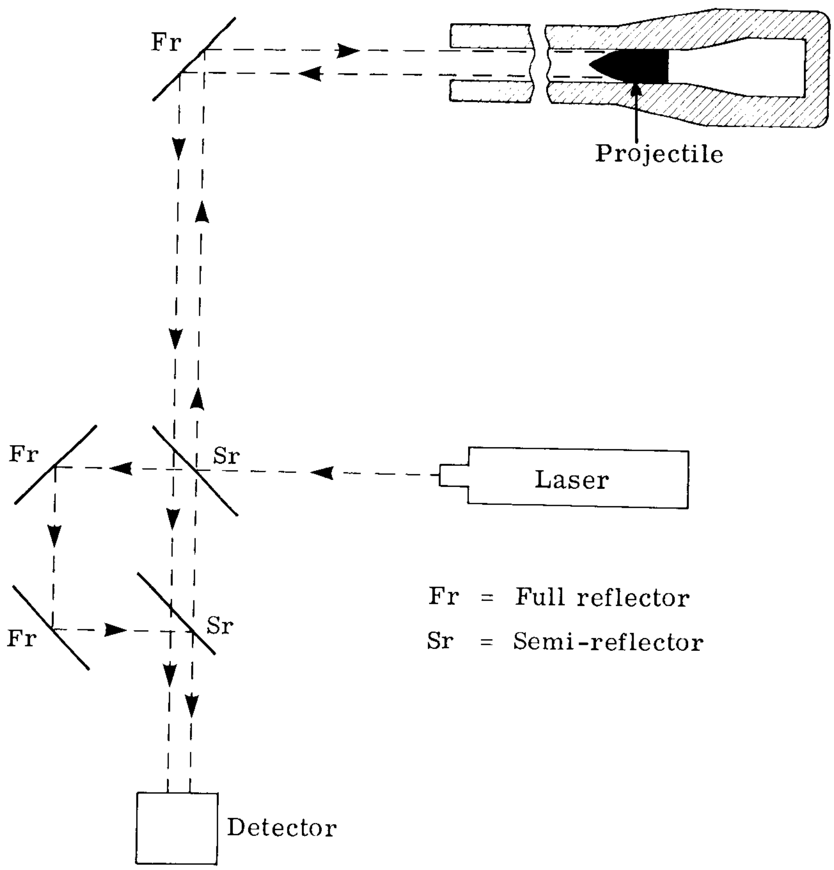

Fig. 7.29 Basic interferometer system

Direct Optical Method

A light beam reflected from the front of a shell can be directed as shown in Fig. 7.30, to a light sensitive detector and so provide a direct optical method of measurement. When the shell is in its start position the light beam strikes the centre of a plane mirror placed on the shell nose; the reflected beam from the shell then passes through an optical processing system. As the shell moves towards the muzzle, the reflected beam path changes and an image moves across the face of the detector device. If the detector consists of a graduated coated piece of glass, and focusing arrangement, then the light intensity picked up by a photo-detector indicates the shell displacement.

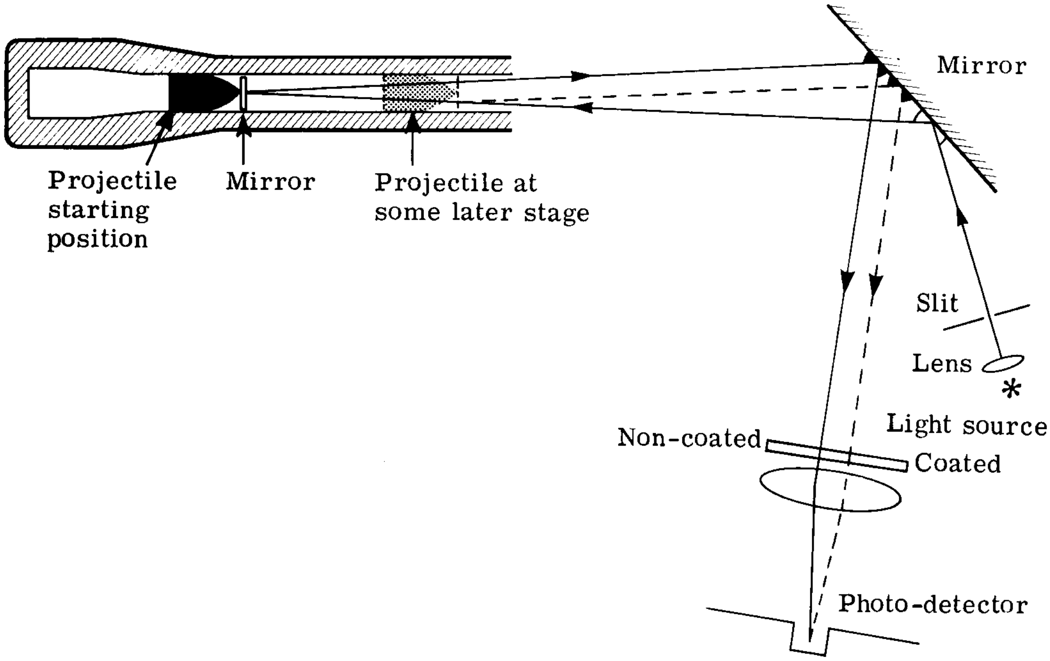

Fig. 7.30 Direct optical method

Bore-Wire Resistance Method

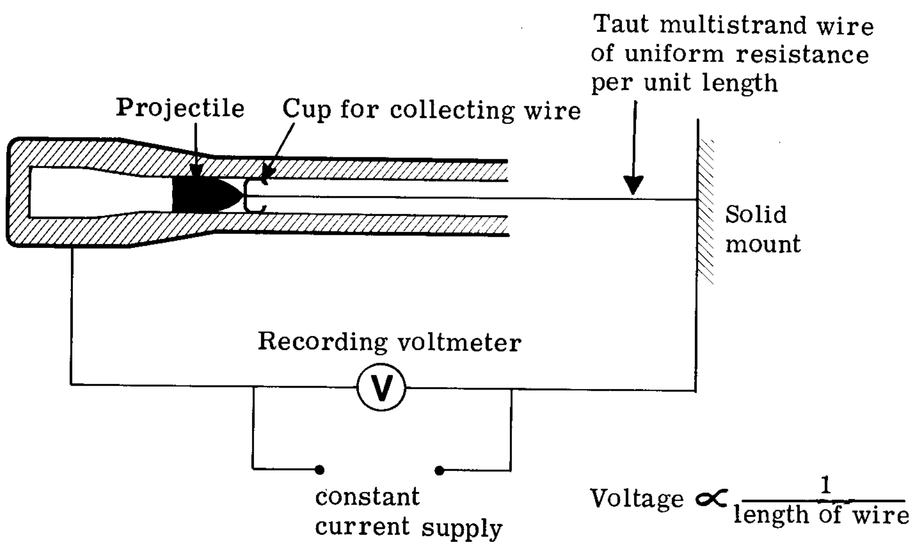

Fig. 7.31 Bore-wire resistance method

In the bore-wire resistance method a wire collecting cup is connected to the nose of the shell. A multistrand bore-wire of constant resistance per-unit-length is connected tautly between the cup and a solid mount. A constant current supply is connected to the bore-wire, with a return path through the shell body and driving band to the gun barrel; the shell acts as a low-resistance moveable terminal. As it moves, the bore-wire is gathered in the cup and the decrease in bore-wire length produces a change in resistance which can be detected as a voltage change at any point between the cup and the current supply.

Barrel Contacts

Insulating probes can be inserted through holes drilled in the barrel wall to make barrel contacts. When the shell makes contact with them an electrical circuit is completed and provides signals for recording the time of arrival. It is very important that the probes are placed at the correct depth to ensure each probe operates at the same relative shell position.

Barrel Strain Gauges

In this technique, strain gauges are mounted circumferentially at a number of positions on the exterior of the barrel. They then produce the required information by acting as 'time of arrival' gauges.

Linear Displacement Transducers

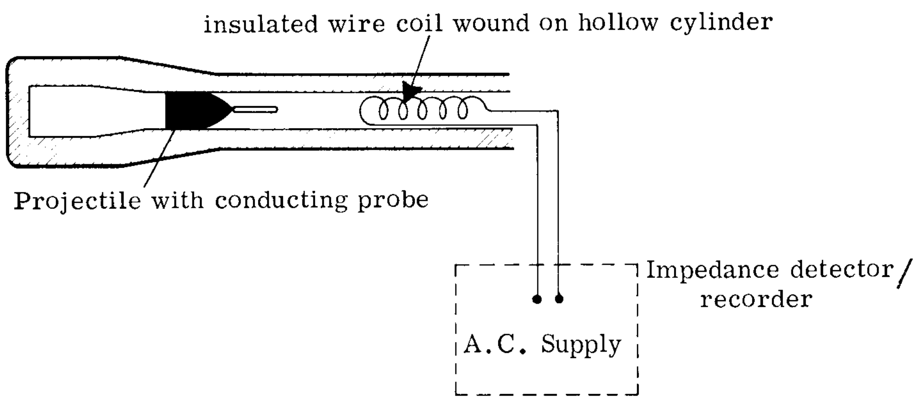

Fig. 7.32 Linear displacement transducers

Linear displacement transducers use a coil assembly wound on a hollow cylinder through which a rod acting as a transformer core moves. As the conducting core is moved through the coil the impedance rises and gives the required measurements.

PROJECTILE ACCELERATION

Accelerometers

All gauges used for in-bore work are deflection-type accelerometers which basically consist of a seismic mass and some, preferably linear, force-resisting process. The level of acceleration is detected by the 'deflection' of the restoring system as it responds to the inertial force on the seismic mass. In Piezoelectric accelerometers the seismic mass is supported by a piezoelectric material. When the crystal is stressed by the mass under acceleration an electric charge is generated which can then be processed using charge-amplifier techniques.

Capacitive Accelerometers

Capacitive accelerometers consist of deflecting a diaphragm replacing the distinct 'mass' and 'spring' sections of other gauges: as a result acceleration may be measured by observing the change in capacitance between the insulated metal diaphragm and the gauge housing.

PROJECTILE ROTATION

The measurement of rotation in the bore can be carried out using optical or microwave methods. In both cases a beam is transmitted from an external source down the barrel where it is returned by special reflectors placed on the nose of the shell.

THERMAL MEASUREMENTS

Thermal measurements related to internal ballistics phenomena are difficult to make and it is not certain whether existing techniques provide sufficient data and accuracy for them to be used with confidence.

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SELF TEST QUESTIONS

QUESTION 1 In ballistics, what is the essential requirement of any photographic system?

Answer ....

QUESTION 2 How is it possible to measure the spin rate of projectiles using the microflash system?

Answer ....

....

....

....

....

....

....

QUESTION 3 For what studies is spark photography used for?

Answer ....

....

....

....

QUESTION 4 What are typical framing rates for
(a) slow speed cine?
(b) medium speed cine?
(c) high speed cine?

Answer (a) ....

(b) ....

(c) ....

QUESTION 5 What photographic system would be particularly useful for studying the interactive process between projectile and target during penetration?

Answer ....

QUESTION 6 What information can yaw sonde provide?

Answer . . . .

. . . .

. . . .

QUESTION 7 What two basic types of gauges are available to measure gas pressure inside a gun barrel?

Answer . . . .

. . . .

QUESTION 8 What system is available for measuring a continuous record of in-bore velocity?

Answer . . . .

ANSWERS ON PAGE 206

Answers to Self Test Questions

Chapter 1

Page 7

QUESTION 1 Leonardo da Vinci.

QUESTION 2 Galileo Galilei.

QUESTION 3 a. Rigid body dynamics.
b. Fluid dynamics.

QUESTION 4 Ballistic pendulum.

QUESTION 5 Drag as a function of the properties of air was recognised.

QUESTION 6 To allow the use of streamlined spun-stabilised projectiles with a consequent increase in range and velocity.

QUESTION 7 Bourne fired the powder in a small metal cylinder and the extent to which the lid rose gave an indication of the "strength" of the powder.

QUESTION 8 The law relating to pressure and density at constant volume for gases.

QUESTION 9 The lack of sophisticated pressure measuring equipment and high speed photography.

QUESTION 10 a. Forensic ballistics.
b. Wound ballistics.

Chapter 2 - Part I

Page 36

QUESTION 1 Propellants do not require atmospheric oxygen for combustion.

QUESTION 2 To achieve location, obturation (sealing) and spin of the projectile.

QUESTION 3 Composition dependent: burning rate constant, pressure index, force constant and co-volume. Shape dependent: ballistic size and form function.

QUESTION 4 Ignition, shot-start, engraving, peak pressure, all-burnt, muzzle exit, maximum velocity.

QUESTION 5 Sustained pressure, late all-burnt, strong muzzle flash and blast, and high but inconsistent muzzle velocity.

QUESTION 6 Very slow burning propellants in relatively large chambers.

QUESTION 7 The surface area increases during burning.

QUESTION 8 Advantage : reduces barrel distortion.
Disadvantage : reduced cooling promotes erosion during repetitive firing.

QUESTION 9 Kinetic energy of projectile = 32% of total energy, and rotational energy = 0.15%; so rotational energy ÷ kinetic energy = $0.15\% \times \frac{100}{32} = 0.5\%$.

QUESTION 10 The propellant gases are cooled by expansion and contact with the bore and chamber walls.

Chapter 2 - Part II

Page 52

QUESTION 1 Burning rate is equal to twice the rate of regression.

QUESTION 2 The total propellant mass and granule shape are retained, but the ballistic size is halved so as to produce the same propellant vivacity.

QUESTION 3 Mass of gas liberated = Cz.

QUESTION 4
a. Remains stationary.
b. Decelerates.

QUESTION 5 The ability to model flame spread during ignition, and pressure oscillations throughout the firing sequence.

QUESTION 6 The Noble-Abel equation only describes the state of a stationary gas; it does not take the velocity and acceleration of the gas into account.

Chapter 3

Page 69

QUESTION 1 Blast shock wave and bottle shock consisting of a barrel shock and Mach disc. These appear in both the precursor and main blast fields, though the main blast field is much more intense.

QUESTION 2 Slight acceleration and yawing of the projectile.

QUESTION 3 The speed of sound.

QUESTION 4 Preflash, primary flash, muzzle glow and intermediate flash.

QUESTION 5 Flash blast.

QUESTION 6 172 dB.

QUESTION 7 The recoil momentum is equal to the total forward momentum, which includes the momentum of the propellant gases.

Chapter 4 - Part I

Page 101

QUESTION 1 45^{O}.

QUESTION 2 $\sin 30^{O} = \frac{1}{2}$, $\therefore$ $V = 2a = 2 \times 340 = 680$ m/s.

QUESTION 3 The form of air resistance which results from the inability of the air flow to return quickly enough to fill the space behind the projectile. This creates a vacuum or suction effect.

QUESTION 4
a. Boattailing.
b. Base bleed.
c. External burning.

QUESTION 5 The transonic zone is the region of greatest instability and air resistance.

QUESTION 6
a. Increased drag.
b. Cross-wind force.
c. Over-turning moment.

QUESTION 7 There is a maximum length of about seven calibres that can be stabilised by spinning alone.

QUESTION 8
a. Wind effects.
b. Gyroscopic drift.
c. Magnus effect.
d. Rotation of the earth.

QUESTION 9 The physical properties of the atmosphere affect the resistance of the air and therefore the range of a projectile.

QUESTION 10 Long range missiles.

QUESTION 11 The rearward gases exhaust with a certain force: this force is reacted against by a thrust in the opposite direction.

Chapter 4 - Part II

Page 135

QUESTION 1 Approximately 5 km.

QUESTION 2 Dynamic similarity.

QUESTION 3 Equality of drag coefficient.

QUESTION 4 Dimples reduce wake caused by boundary layer separation.

QUESTION 5 The point on the axis of an axisymmetric body where the resultant force appears to act, about which therefore the net moment is zero.

QUESTION 6 When the body is neutrally stable, that is, when the over-turning moment coefficient derivative is zero.

QUESTION 7

$$\begin{aligned} N &= \pi V/nr \text{ rad/sec} \\ &= V/2nr \text{ revs/sec} \\ &= V/nd \text{ revs/sec} \end{aligned} \qquad \text{and}$$

$$n = 30.48/0.556 = 54.82 \qquad \text{therefore}$$

$$N = 3250 \text{ revs/sec} \qquad \text{alternatively,}$$

$$N = V/3048$$

$$= 3250 \text{ revs/sec.}$$

QUESTION 8 The drift increases as the angle of projection is increased.

Chapter 5 - Part I and Part II

Page 158

QUESTION 1

a. Angle of attack.
b. Strike velocity.
c. Target configuration and composition.
d. Projectile configuration and composition.

QUESTION 2 Penetration may be defined as the entrance of a missile into a target without completing its passage: perforation is the complete piercing of the target by the projectile.

QUESTION 3
a. Impact phase.
b. Penetration.
c. Perforation.

QUESTION 4
a. Fracture.
b. Spalling.
c. Scabbing.
d. Petalling.
e. Fragmentation.
f. Plugging.

QUESTION 5
a. High density core.
b. Discarding sabot.
c. Long thin rod for optimum penetration.
d. Low drag profile.
e. Spin stabilisation.
f. High muzzle velocity.

QUESTION 6
a. Higher muzzle velocity
b. Greater penetration

QUESTION 7
a. In destruction of buildings and structural damage in general.
b. Open environment.

QUESTION 8
Through spall and by physiological and pyschological effects against the tank crew by pressure, temperature and flash.

QUESTION 9
HESH.

QUESTION 10
a. Empirical.
b. Analytical.
c. Experimental.
d. Numerical.

Chapter 6

Page 170

QUESTION 1
Absorption of kinetic energy imparted by missile.

QUESTION 2
Elasticity and density of tissue.

QUESTION 3
The transfer of projectile momentum to surrounding tissues.

QUESTION 4
No. For the same amount of total energy expended, the design of the bullet can make profound differences in its effect on tissues.

QUESTION 5
a. Resist penetration of the projectile.
b. Dissipate most of the kinetic energy of the shot.
c. Satisfy the restraints of bulk and weight.

QUESTION 6 Advantages:
a. Effective against high velocity bullets.
b. Able to withstand close and repeated shots.

Disadvantages:
a. Spalling.
b. Possible effects of cavitation.
c. Heavy (although light low-alloy steels are available at extra costs).

QUESTION 7 Advantages:
a. Extremely hard.
b. Lightweight.

Disadvantages:
a. Any one ceramic tile cannot accept more than one hit.
b. Expensive.

QUESTION 8 The heavy weave of the material deforms the nose of the projectile, so preventing the mushroomed bullet from forcing its way through the mesh of fibres.

Chapter 7

Page 199

QUESTION 1 To "freeze" the motion.

QUESTION 2 Lines are painted longitudinally on the projectile body which is photographed in flight in the light of two microflash lamps spaced a short distance apart down range. The time interval between the two flashes is recorded and the rotation of the shell during this interval can be measured from the photograph.

QUESTION 3
a. Armour protection and penetration.
b. Yaw measurement.
c. Air resistance.
d. Aerodynamic performance of a particular shape of projectile.
e. Pressure disturbances in surrounding airflow.

QUESTION 4
a. 24 p.p.s.
b. 10,000 p.p.s.
c. Several million p.p.s.

QUESTION 5 Flash radiography (X-ray shadowgraph).

QUESTION 6 A measure of yaw and spin rate, precision and stability, for shells in free flight for most of the trajectory.

QUESTION 7
a. Crusher gauge.
b. Piezoelectric gauge.

QUESTION 9 Interferometer.

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